



Influence of a Strong Electromagnetic Wave on the Nonlinear Absorption coefficient of Strong Electromagnetic Waves Caused by Electrons Confined in Cylindrical Quantum Wires

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ABSTRACT

The nonlinear absorption coefficient of a strong electromagnetic wave caused by electrons confined in cylindrical quantum wires is theoretically studied by using the quantum kinetic equation for electrons. Analytic expressions of the nonlinear absorption coefficient of a strong electromagnetic wave caused by electrons confined in a cylindrical quantum wire with a parabolic potential for electron-optical phonon scattering is obtained, that the dependence of the nonlinear absorption coefficient on the intensity E_0 and the frequency Ω of the external strong electromagnetic wave is strongly and nonlinearly. The analytic expressions are numerically calculated and discussed for a GaAs/GaAsAl quantum wire. The results are compared with those for normal bulk semiconductors and quantum wells to show the differences.

KEYWORDS: Cylindrical Quantum Wire, Nonlinear Absorption, Electron-Optical Phonon Scattering.

1. INTRODUCTION

In recent times, the optical properties in bulk semiconductors, as well as low-dimensional systems, have also been investigated [1-7]. In those articles, the linear absorption of a weak electromagnetic wave was considered in normal bulk semiconductors [1], in two dimensional systems [2-4] and in quantum wires [5]; the nonlinear absorption of a strong electromagnetic wave (EMW) was considered in normal bulk semiconductors [6] and quantum wells [7]. However, in cylindrical quantum wires, the nonlinear absorption of a strong EMW still opens for studying.

In one dimensional systems, the motion of electrons is restricted in two dimensions, so they can flow freely in one dimension. The confinement of electrons in these systems changes the electron mobility remarkably. This results in a

number of new phenomena, which concern a reduction of the sample dimensions. These effects, for example, electron-phonon interaction and scattering rates [8, 9] and dc electrical conductivity [10, 11], differ from those in bulk semiconductors, as well as two-dimensional, and the nonlinear absorption of strong EMW is not an exception. The nonlinear absorption a strong EMW in a quantum wire will be different from the nonlinear absorption a strong EMW in bulk semiconductors and two-dimensional. In this paper, we use the quantum kinetic equation for electrons to theoretically study the nonlinear absorption coefficient of a strong EMW by electrons confined in a cylindrical quantum wire (CQW) with a parabolic potential. The problem is considered for electron-optical phonon scattering. Numerical calculations were carried out for specific GaAs/GaAsAl quantum wires.

2. THE NONLINEAR ABSORPTION COEFFICIENT OF A STRONG EMW IN A CQW

Experimental We consider a wire of GaAs with a circular cross section with a radius R and a length L_z embedded in AlAs. The carriers (electrons) are assumed to be confined by symmetric parabolic potential of the form

$$V = \frac{1}{2}m\omega_0^2r^2, \quad (1)$$

where m is the effective mass of electron, ω_0 is the effective frequency of potential well. The total wave function of electrons in cylindrical coordinates (r, ϕ, z) and the electron energy spectrum obtained directly from solving Schrodinger equation and can be written as

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$$\psi_{n,\ell,\vec{p}}(r) = \frac{e^{i\vec{p}z}}{\sqrt{L_z}} \sqrt{\frac{2n!}{(n+|\ell|)!}} \frac{1}{a_0} e^{-\frac{r^2}{2a_0^2}} \left(\frac{R}{a_0}\right)^{|\ell|} \left(\frac{r^2}{a_0^2}\right)^{|\ell|} L_n^{|\ell|}\left(\frac{r^2}{a_0^2}\right);$$

$$\varepsilon_{n,\ell}(\vec{p}) = \frac{p_z^2}{2m} + \omega_0 (2n + |\ell| + 1), \quad (2)$$

where $n = 0, \pm 1, \pm 2, \dots$ is the azimuthal quantum number, $\ell = 1, 2, 3, \dots$ is the radial quantum number, $\vec{p} = (0, 0, p_z)$ is the electron wave vector (along the wire's z axis), $L_n^{|\ell|}(x)$ is a generalized lagrange polynomial, $a_0 = 1/\sqrt{m\omega_0}$ (in this paper, we select $\hbar = 1$).

The Hamiltonian of the electron-optical phonon system in a cylindrical quantum wire in the presence of a laser field, $\vec{E}(t) = \vec{E}_0 \sin(\Omega t)$, can be written as

$$H = \sum_{n,\ell,\vec{p}} \varepsilon_{n,\ell}(\vec{p} - \frac{e}{c} \vec{A}(t)) a_{n,\ell,\vec{p}}^+ a_{n,\ell,\vec{p}} + \sum_{\vec{q}} \omega b_{\vec{q}}^+ b_{\vec{q}} + \sum_{n,\ell,n',\ell',\vec{p},\vec{q}} C_{\vec{q}} I_{n,\ell,n',\ell'}(\vec{q}) a_{n,\ell,\vec{p}+\vec{q}}^+ a_{n',\ell',\vec{p}} (b_{\vec{q}} + b_{-\vec{q}}^+), \quad (3)$$

where e is the electron charge, c is the velocity of light, $\vec{A}(t) = \frac{c}{\Omega} \vec{E}_0 \cos(\Omega t)$ is the vector potential, \vec{E}_0 and Ω is the intensity and the frequency of the EMW, $a_{n,\ell,\vec{p}}^+$ ($a_{n,\ell,\vec{p}}$) is the creation (annihilation) operator of an electron, $b_{\vec{q}}^+$ ($b_{\vec{q}}$) is the creation (annihilation) operator of a optical phonon for a state having wave vector \vec{q} , ω is the frequency of a optical phonon, and $C_{\vec{q}}$ is the electron-optical phonon interaction constant, it can be taken as [1-3,6]

$$|C_{\vec{q}}|^2 = e^2 \omega (1/\chi_\infty - 1/\chi_0) / 2\varepsilon_0 q^2 V, \quad (4)$$

where ε_0 is the permittivity of free space, V is the normalization volume, and χ_∞ and χ_0 are the high-frequency dielectric constants and low-frequency dielectric constants, respectively. $I_{n,\ell,n',\ell'}(\vec{q})$ is the electron form factor and can be written as

$$I_{n,\ell,n',\ell'}(q_\perp) = \frac{2}{R^2} \int_0^R \psi_{n,\ell}(r) e^{iqr} \psi_{n',\ell'}^*(r) r dr. \quad (5)$$

The carrier current density $\vec{j}(t)$ and the nonlinear absorption coefficient α of a strong electromagnetic wave α take the form [6,7,12]

$$\vec{j}(t) = \frac{e}{m} \sum_{n,\ell,\vec{p}} (\vec{p} - \frac{e}{c} \vec{A}(t)) n_{n,\ell,\vec{p}}(t);$$

$$\alpha = \frac{8\pi}{c\sqrt{\chi_\infty E_0^2}} \langle \vec{j}(t) \vec{E}_0 \sin \Omega t \rangle_t \quad (6)$$

where $n_{n,\ell,\vec{p}}(t)$ is the electron distribution function, $\langle X \rangle_t$ means the usual thermodynamic average of X at moment t , and χ_∞ is the high-frequency dielectric constant. In order to establish analytical expressions for the nonlinear absorption coefficient of a strong EMW by electrons confined in a CQW, we use the quantum kinetic equation for particle number operator of an electron

$$n_{n,\ell,\vec{p}}(t) = \langle a_{n,\ell,\vec{p}}^+ a_{n,\ell,\vec{p}} \rangle_t$$

$$i \frac{\partial n_{n,\ell,\vec{p}}(t)}{\partial t} = \langle [a_{n,\ell,\vec{p}}^+ a_{n,\ell,\vec{p}}, H] \rangle_t \quad (7)$$

From Eq.(7), using the Hamiltonian in Eq. (4) and realizing calculations, we obtain quantum kinetic equation for the electrons confined in a CQW. Using the first-order tautology approximation method (the approximation is applied for a similar exercise in bulk semiconductors [12] and quantum wells [7]) to solve this equation, we obtain the expression of electron distribution function, $n_{n,\ell,\vec{p}}(t)$:

$$n_{n,\ell,\vec{p}}(t) = - \sum_{\vec{q},n',\ell'} |C_{\vec{q}}|^2 |I_{n,\ell,n',\ell'}(\vec{q})|^2 \sum_{k,l=-\infty}^{\infty} J_k\left(\frac{e\vec{E}_0\vec{q}}{m\Omega^2}\right) J_{k+l}\left(\frac{e\vec{E}_0\vec{q}}{m\Omega^2}\right) \frac{1}{l\Omega} e^{-il\Omega t} \times$$

$$\times \left[- \frac{\bar{n}_{n,\ell,\vec{p}}(N_{\vec{q}}+1) - \bar{n}_{n',\ell',\vec{p}+\vec{q}} N_{\vec{q}}}{\varepsilon_{n',\ell',\vec{p}+\vec{q}}(\vec{p}+\vec{q}) - \varepsilon_{n,\ell}(\vec{p}) + \omega - k\Omega + i\delta} - \frac{\bar{n}_{n,\ell,\vec{p}} N_{\vec{q}} - \bar{n}_{n',\ell',\vec{p}+\vec{q}}(N_{\vec{q}}+1)}{\varepsilon_{n',\ell',\vec{p}+\vec{q}} - \varepsilon_{n,\ell,\vec{p}} - \omega_{\vec{q}} - k\Omega + i\delta} \right.$$

$$\left. + \frac{\bar{n}_{n',\ell',\vec{p}-\vec{q}}(N_{\vec{q}}+1) - \bar{n}_{n,\ell,\vec{p}} N_{\vec{q}}}{\varepsilon_{n,\ell}(\vec{p}) - \varepsilon_{n',\ell',\vec{p}-\vec{q}} + \omega - k\Omega + i\delta} + \frac{\bar{n}_{n,\ell,\vec{p}-\vec{q}} N_{\vec{q}} - \bar{n}_{n,\ell,\vec{p}}(N_{\vec{q}}+1)}{\varepsilon_{n,\ell}(\vec{p}) - \varepsilon_{n',\ell'}(\vec{p}-\vec{q}) - \omega - k\Omega + i\delta} \right], \quad (8)$$

where $N_{\vec{q}}$ ($\bar{n}_{n,\ell,\vec{p}}$) is the time independent component of the phonon (electron) distribution function, $J_k(x)$ is the Bessel function, and the quantity δ is infinitesimal and appears due to the assumption of an adiabatic interaction of the electromagnetic wave. We

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insert the expression for $n_{n,\ell,\bar{p}}(t)$ into the expression for $\vec{j}(t)$ and then insert the expression for $\vec{j}(t)$ into the expression for α in Eq.(6). Using the properties of Bessel function and realizing the calculations, we obtain the nonlinear absorption coefficient of a strong EMW by confined electrons in a CQW as

$$\alpha = \frac{8\pi^3 e^2 \Omega k_b T}{c \sqrt{\chi_\infty} E_0^2 \varepsilon_0 V_0} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0} \right) \sum_{n,\ell,n',\ell'} \sum_{\bar{q},\bar{p}} |I_{n,\ell,n',\ell'}(q)|^2 \frac{1}{q^2} \sum_{k=-\infty}^{\infty} [\bar{n}_{n,\ell,\bar{p}} - \bar{n}_{n',\ell',\bar{p}+\bar{q}}] \times \\ \times k J_k^2 \left(\frac{e E_0 \bar{q}}{m \Omega^2} \right) \delta(\varepsilon_{n',\ell',\bar{p}+\bar{q}} - \varepsilon_{n,\ell,\bar{p}} + \omega - k\Omega), \quad (9)$$

where $\delta(x)$ is the Dirac delta function. We only consider the absorption close to its threshold because in the rest case (the absorption far away from its threshold) α is very smaller. In this case, the condition $|k\Omega - \omega| = \bar{\varepsilon}$ must be satisfied. Therefore, we can not ignore the presence of vector \bar{p}_z in the formula of the δ function. We restrict the problem to the case of one photon absorption and consider the electron gas to be non-degenerate:

$$\bar{n}_{n,\ell,\bar{p}} = n_0^* \exp\left(-\frac{\varepsilon_{n,\ell,\bar{p}}}{k_b T}\right), \quad \text{with} \quad n_0^* = \frac{n_0 (e\pi)^2}{V_0 (m_0 k_b T)^2} \quad (10)$$

where m_0 is the mass of free electron, and k_b is the Boltzmann constant. Using the Bessel function, and the energy spectrum of an electron in a CQW, we have the an explicit formula of the nonlinear absorption coefficient in a CQW for the case of the absorption close from its threshold:

$$\alpha = \frac{\sqrt{2\pi} e^4 n_0^* (k_b T)^{3/2}}{4c \varepsilon_0 \sqrt{m \chi_\infty} \Omega^3 V_0} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0} \right) \sum_{n,\ell,n',\ell'} |I_{n,\ell,n',\ell'}(q)|^2 \left[\exp\left\{ \frac{1}{k_b T} (\omega - \Omega) \right\} - 1 \right] \times \\ \times \exp\left\{ \frac{1}{k_b T} B_1 \right\} \left[1 + \frac{3e^2 E_0^2 k_b T}{8m\Omega^4} \left(1 + \frac{B_1}{2k_b T} \right) \right] + [\omega \rightarrow -\omega] \quad (11)$$

where $B_1 = \omega_0 (2n' - 2n + |\ell'| - |\ell|) + \omega - \Omega$. From the analytic expression for the nonlinear absorption coefficient of a strong EMW caused by electrons confined in CQWs with a parabolic potential (Eq. 11), we can see that when the term proportional to the quadratic intensity of the EMW (E_0^2) tend toward zero, the nonlinear result will become a linear result.

3. NUMERICAL RESULTS AND DISCUSSIONS

In order to clarify the results that have been obtained, in this section, we numerically calculate the nonlinear absorption coefficient of a strong EMW for a *GaAs / GaAsAl* CQW. The nonlinear absorption coefficient is considered as a function of the intensity E_0 and the energy of the strong EMW, the temperature T of the system, and the parameters of the CQW. The parameters used in the numerical calculations [6,13] are $\xi = 13.5 \text{ eV}$, $\rho = 5.32 \text{ gcm}^{-3}$, $v_s = 5378 \text{ ms}^{-1}$, $\varepsilon_0 = 12.5$, $\chi_\infty = 10.9$, $\chi_0 = 13.1$, $m = 0.066m_0$, m_0 being the mass of free electron, $\hbar\omega = 36.25 \text{ meV}$, $k_b = 1.3807 \times 10^{-23} \text{ J/K}$, $n_0 = 10^{23} \text{ m}^{-3}$, $e = 1.60219 \times 10^{-19} \text{ C}$, $\hbar = 1.05459 \times 10^{-34} \text{ J.s}$.

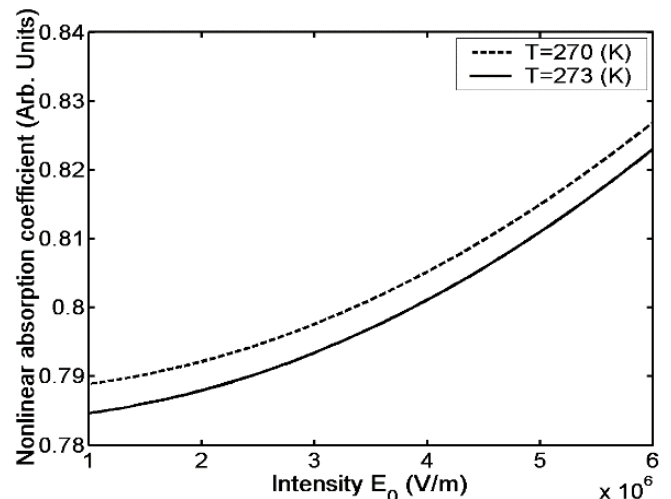


Figure 1: Dependence of α on E_0 (The absorption close from its threshold)

Figure 1 presents the dependence of the nonlinear absorption coefficient α on the intensity E_0 of EMW. This dependence shows that the nonlinear absorption coefficient α is descending when the intensity E_0 of EMW increases.

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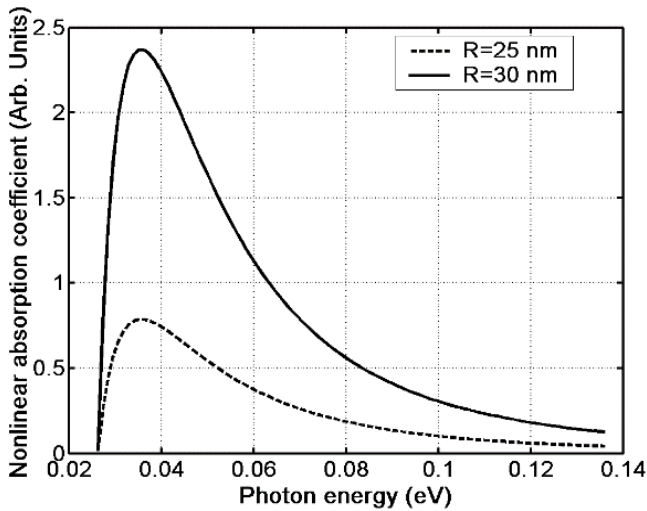


Figure 2: Dependence of α on EMW energy (The absorption close from its threshold)

Figure 2 presents the dependence of α on the EMW energy at different values of the radius of wire. It is seen that α has the same maximum values (peaks) at $\Omega \equiv \omega$. The EMW energy at which α has a maximum are not changed as the radius of wire is varied. This means that α depends strongly on the frequency Ω of the EMW and resonance conditions are determined by the EMW energy. In addition, it can be seen from figures 1-2 that different from normal bulk semiconductors [6] and quantum wells [7], the nonlinear absorption coefficient α in quantum wires is bigger. This is explained that when electrons is confined in a quantum wire, the electron energy spectrum continue to be quantized. So the absorption of a strong electromagnetic wave is better. This fact is also reflected in the expressions of the nonlinear absorption coefficient (Eq. 11). Besides the sum over quantum n (as in quantum well), the expressions of the nonlinear absorption coefficient in quantum wire have the sum over the quantum number ℓ .

4. CONCLUSION

In this paper, we have obtained analytical expression for the nonlinear absorption of a strong EMW by confined electrons in CQW for the case electron-optical phonon scattering and numerically calculate for a *GaAs / GaAsAl* quantum wire.

The analytical results and numerical results obtained for a *GaAs / GaAsAl* CQW show that α depends strongly and nonlinearly on the intensity E_0 and frequency Ω of the external strong EMW, the temperature T of the system and the radius R of wires. This dependence has differences in comparison with that in normal bulk semiconductors [6] and two dimensional systems [7]. Compared with normal bulk semiconductors [6] and two dimensional systems [7], the nonlinear absorption coefficient in quantum wire much larger. The results show a geometrical dependence of α due to the confinement of electrons in one dimensional systems. In addition, from the analytic results, we see that when the term in proportion to quadratic the intensity of the EMW (E_0^2) (in the expressions of the nonlinear absorption coefficient of a strong EMW) tend toward zero, the nonlinear result will turn back to a linear result.

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