



## Improve the Ability to Use Mathematical Representation for Junior High School Students in Vietnam

Vu Thi Binh

Thai Nguyen University – Lao Cai Campus  
Group 13, Binh Minh Ward, Lao Cai City, Lao Cai Province, Vietnam

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Corresponding Author:  
Vu Thi Binh

### ABSTRACT

The world's tendency to develop competence in secondary education and the need to reform secondary education in Vietnam today aim to reach UNESCO's four pillars of learning in the 21st century learning to know, learning to do, learning to be, and learning to live together. This article aims to create opportunities for students to form and develop their mathematical representation competencies through teaching Mathematics in junior high schools in Vietnam. By surveying 1390 students in 15 secondary schools about the actual situation of learning mathematical representations, to analyze, compare and propose solutions to improve mathematical representation competence for students. The results showed many opportunities for students to observe and compare, help students understand and use different mathematical representations when expressing similar relationships, and flexibility in mathematical operations in the learning process.

**KEYWORDS:** Mathematical representation competence, mathematical teaching, General school.

### INTRODUCTION

Today, the overview that education to form mathematical competence for students through activities and learning activities has been confirmed by many mathematical educators. According to PISA, individuals who use their mathematical knowledge within various contexts need to possess several mathematical competencies. To identify and examine these competencies, PISA has decided to use eight distinctive mathematical competencies relevant and meaningful across all education levels. *Representation competence is a fundamental competency that is critically important to mathematical literacy is the capacity to successfully use and manipulate various representations of mathematical objects and situations. This may include such representations as graphs, tables, charts, photographs, diagrams, text, and algebraic and other symbolic mathematical expressions. Central to this competence is understanding and using interrelationships among these different representations* (OECD 2009).

When the American cognitive psychologist J. Bruner focused on the cognition of children, as well as representative thinking, he pointed out that it is possible to divide representation into the following three classifications, which describe the sequential development stages of representation: (E) Enactive representation; (I) Iconic representation; (S) Symbolic representation (Nakahara

2007). Accordingly, there are three learning activities responding to the students (1). Object description (manipulation); (2). Ionization and (3). Symbolization (sign) (Ngo 2011). This ideal became the key factor in teaching mathematics; the teachers always know that the students have to start with their specific experience, switching to icons, pictures, and finally understanding the abstract symbols.

Lesh, Landau, and Hamilton (1983) found five kinds of representations that are useful for mathematical understanding: (a) real-life experiences, (b) manipulative models, (c) pictures or diagrams, (d) spoken words, and (e) written symbols. Based on the researches by J. Bruner and Lesh, Tadao proposes the broad organization of representational modes in mathematics education into the following five categories: S2. Symbolic representation; S1. Linguistic expression; I. Illustrative representation; E2. Manipulative presentation; E1. Realistic picture (Nakahara 2007). In 2000, the National Council of Teachers of Mathematics (NCTM) determined that representation is 1 of 5 Process Standards of School Mathematics. Then, Mathematical Representation has been studied in more detail, concretization into Representation Standard for School Mathematics: Prekindergarten through Grade 12, which is the compulsory standard in teaching and evaluating

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school mathematics in America and some other countries in the world (NCTM 2000).

In Vietnam, the General education curriculum 2018 identifies five mathematical competencies: *Mathematical thinking and reasoning*; *Modeling*; *Problem-solving*; *Mathematical communication*; *Use tools and means of learning math*. The research results mentioned above show that mathematical representation plays an essential role in forming and developing mathematical competencies for students. Using mathematical expressions helps students become better when thinking, reasoning, modeling, communicating, and solving math problems. This study focuses on identifying opportunities for students to create and use representations to search for mathematical solutions flexibly, creatively, and effectively, contributing to math competencies.

### MATERIALS AND METHODS

In 2020, survey 1390 junior high school students in Lao Cai province (Vietnam) through questionnaires about using mathematical representations and learning that students use such mathematical expression; attending the mathematical classes in Junior high school, examining notebooks, mathematical tests. All of these methods showed that students have many difficulties in representing mathematical content. The students are embarrassed to use the representations such as photographs, charts, signs, symbols, so it is difficult for them to find mathematical solutions in learning and real life. Many teachers have no effective measures to organize activities in mathematical representation for the students. This causes the fact that when learning mathematics, the students are not proactive, self-confident, and flexible in using mathematics in dealing with practical issues in real life. The following tables are illustrations for the above judgments.

**Table 1:** School students' competence to use symbols, photographs, charts, diagrams, graphs (representations)

Content	Competence							
	Very good		good		Average		Poor	
	Q.ty	%	Q.ty	%	Q.ty	%	Q.ty	%
(a) Use representations to organize, record, and communicate mathematical ideas;	182	13,1	279	20,1	678	48,8	251	18,1
(b) Create representations to organize, record, and communicate mathematical ideas;	154	11,1	258	18,6	692	49,8	286	20,6
(c) Select, apply, and translate among mathematical representations to solve problems	138	9,9	235	16,9	712	51,2	305	21,9
(d) Use representations to model and interpret physical, social, and mathematical phenomena.	126	9,1	225	16,2	723	52,0	316	22,7

The content of Table 1 shows that the ability uses symbols, drawings, charts, diagrams, graphs (or mathematical representations) is not high. The percentage of students who know how to use mathematical expressions is at a weak level, 18.1%. Most students reach the average level, 48.8%. The percentage of students achieving good and well is 20% and 13%, respectively. Most of the remaining criteria are

good and well at a much lower level. In particular, in criterion d), 74.7% of students in the average and weak group, 9% are good and are not well. These results reflect that junior high school students are not proficient in using mathematical symbols to model problems, seek solutions and solve practical problems in mathematical tools.

**Table 2:** Students learn to use symbols, photographs, charts, diagrams, tables (representations) in mathematical classes

Content	Frequency							
	Very often		Often		Rarely		Never	
	Q.ty	%	Q.ty	%	Q.ty	%	Q.ty	%
(a) Observe, imitate the use of representations of the teachers	1113	80,1	235	16,9	42	3,0	0	0,0
(b) Observe, imitate the use of representations of other friends	672	48,3	385	27,7	212	15,3	121	8,7
(c) Select and create the representations, think to find mathematical solutions	302	21,7	732	52,7	313	22,5	43	3,1
(d) Select and create the representations to represent the mathematical solutions	56	4,0	162	11,7	779	56,0	393	28,3

The content of Table 2 gives another comment on the current situation of junior high school students learning how to use representations in math lessons. It shows that over 80% of students often observe and follow their teacher to display and use mathematical models (Criterion a). Therefore, the teacher has a direct impact on students for the mathematical representation skills. If the teacher rarely uses mathematical representations, then her students may not also use the ones effectively. Table 2 also shows that a large number of students seldom learn from their friends how to use representations. Some students explained that the way their friends used drawings is different and may be inaccurate. Obviously, in teaching math, teachers need to encourage students to use mathematical representations flexibly and creatively. There are 84% of students who seldom or never self-represent representations to show mathematical solutions. Hence, it is necessary has to research and find solutions to improve the formation and development of mathematical representation skills for students in teaching mathematics at junior high school.

**RESULTS AND DISCUSSION**

In primary education, during the process of learning mathematics, the students have got acquainted with and commonly applied the visual, mathematical representations (chart of perpendicular lines, specific objects, images) to express any connections, relationships, things when mathematical operations or formulas are formed, in solving the mathematical literacy, problems of finding two numbers with two conditions. In junior high school, the role of mathematical representation is exploited more profoundly and diversely to find out any mathematical rules, connections, or relationships. The students shall use not only the chart of perpendicular lines, illustrator but also plane figures, cubes, diagrams, charts, tables, graphs..., allowing generalization of standard rules and abstract relationships; then, improving and developing the students' capacity of applying the mathematical representations to the research and explanation of phenomena in life.

Based on the above analysis, this study proposes cases of using mathematical representations for students in teaching lower secondary math as follows:

(1) *Through cognition activities and understanding mathematical contents of mathematical representations accurately, logically, and systematically*

Scientific history shows that mathematical theories' logical structure and development have increasingly depended on mathematical symbols and improved such characters since a definite development period. The system of such characters in mathematics in junior high school has been more and more prosperous and diverse, which may abstract and generalize a mathematical model or relationship under rules. Through the teaching activities to form the

mathematical symbols, the students can read, write, understand the meanings and structure of grammar, identify and express mathematical representations, then help the students understand and use accurately and effectively symbols in learning mathematics.

**Example 1.** Learning function graph  $y = f(x)$ , the students shall identify the way of representing the respective  $(x; y)$  of function  $y = ax$  ( $a \neq 0$ ) on the coordinate plan through the implementation of the following activities:

- (i). Write collection of respective pairs  $(x; y)$  of function  $y = f(x)$ ;
- (ii). Draw coordinate system  $Oxy$ ;
- (iii). Determine points of which coordinate is  $(x;y)$  on the coordinate plane;
- (iv). Introduce name of representation: function graph  $y = f(x);...$

Since then, when a point on the coordinate plane is given, the students may know the coordinate of such a point and check whether such a point is on any function graph; it means that the students can solve the given mathematical representation.

Other than the representations of terminologies, mathematical symbols, the students need to understand the correct meanings of symbolic representations, charts, or mathematical models.

**Example 2.** (Grade 6) The following chart is suggestive, resulting in the fundamental property of fraction:



The correct understanding of such mathematical representations is crucial for the students during mathematical cognition. The teachers need to apply the mathematical representations regularly: tables, line chart, Venn diagram, mind map (not teaching explicitly in the syllabus), and explain such representations to help the students understand and use reason in the study.

(2). *Through linking, transformation, or creation of mathematical representations suitable to specific situations or contexts.*

In teaching mathematics in junior high school, symbols, drawings, diagrams, tables are usually used. Therefore the students can develop and deepen their understanding of concepts and mathematical relations when creating, comparing, and using different representations. They help reduce the abstraction of mathematics, make formulas, mathematical transformations closer to the students' perceptions.

According to J. Piaget, at different age stages, there is the other type of learning action: the learning activities of

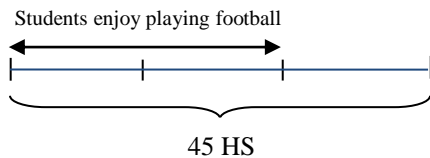
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students in the primary school (from 7-11 years old) usually begins with material, practical action; the students in the junior high school and high school act on objects as symbols, clauses, diagrams, models, etc... In teaching mathematics in junior high school, the activities related to linking, transformation, or creation of the mathematical representations are consistent with the specific and meaningful situations, contexts, make the transition from the *material, practical action* in the mathematical cognition of students to *act on the objects as a symbol, clause, diagram, model, etc.*

**Example 3.** (Grade 6) Exercise: Find the fractional value of a given number.

Considering example: Class 6A has 45 students, of which  $\frac{2}{3}$  of total students enjoy playing football, 60% students enjoy kicking a shuttlecock,  $\frac{2}{9}$  students enjoy playing table tennis and  $\frac{4}{15}$  enjoys playing volleyball. Calculate the number of class 6A enjoying playing football, kicking a shuttlecock, playing table tennis, and volleyball.

The students may be organized to use, link, transform and create the mathematical representations during cognition as follows:

Teacher – Ask	Students- Think and answer
1. Use a line segment diagram to represent the number of class 6A enjoying playing football?	1. 
2. Based on the diagram, calculate the number of students enjoying playing football?	2. The number of students who enjoy playing football: $(45:3).2 = 30$ (students).
3. Write $(45:3).2$ in the form of multiplication (of 45 and $\frac{2}{3}$ )?  - Thence to find out $\frac{2}{3}$ of 45?	3. $(45:3).2 = 45 \cdot \frac{2}{3}$  - Get 45 multiplied with $\frac{2}{3}$ (write: $45 \cdot \frac{2}{3}$ )
4. Similarly, calculate the number of students enjoying kicking a shuttlecock, playing table tennis, volleyball?  (60% of 45; $\frac{2}{9}$ of 45; $\frac{4}{15}$ of 45)	4. The number of students enjoys: - kicking a shuttlecock: $45 \cdot 60\% = 45 \cdot \frac{60}{100} = 27$ (students) - playing table tennis: $45 \cdot \frac{2}{9} = 10$ (students) - playing volleyball: $45 \cdot \frac{4}{15} = 12$ (students)
5. To find $\frac{m}{n}$ a given $b$ , how do we do it?	5. Get $b$ multiplied with $\frac{m}{n}$ ( $b \cdot \frac{m}{n}$ )
General: "To find $\frac{m}{n}$ of a given $b$ , we calculate $b \cdot \frac{m}{n}$ ( $m, n \in N, n \neq 0$ )"	

**Comment:** The mathematical representation activities inhere consist of: *Creation of representation* (a line segment

diagram); *Linking of representation*: From the line segment diagram representation, we find out the symbolic mathematical representation (corresponding to the number of students who enjoy playing football):  $(45:3).2$ ;

*Transformation of representation*:  $(45:3).2 = 45 \cdot \frac{2}{3}$ ;

*Creation of symbolic representation for the general rule*: "To

find  $\frac{m}{n}$  given  $b$ , we calculate  $b \cdot \frac{m}{n}$  ( $m, n \in N, n \neq 0$ )."

Thus, the students thoroughly understand the meaning of the language and symbolic representations in the general rule and practice.

(3). *Through selection, transition activities of the mathematical representations during cognition, practice, memorization, and mathematical communication*

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In teaching mathematics, selection and transition activities of the mathematical representations are usually done. However, purposeful activities allow the students to select and transit the mathematical models in cognition, practice, memorization, and mathematical communication,

which should be paid more attention to. There are many opportunities for the students to select different mathematical representations for the same content in the mathematics learning process.

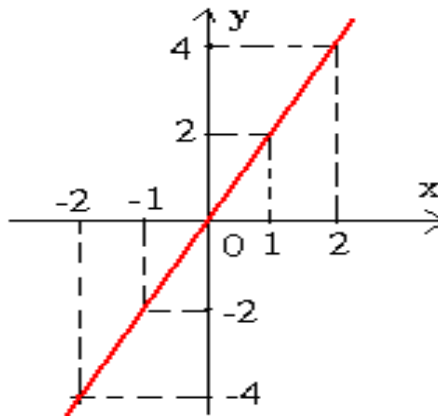
**Example 4.** (Grade 7) To represent the function  $y = 2x$ , depending on the use purpose, the students can select the following representations:

Representation by the formula:

$$y = 2x$$

Representation as a table and representation by a diagram (figure 1).

x	-2	-1	0	1	2	...
Y=2x	-4	-2	0	2	4	...



**Figure 1:** representation by a diagram

In teaching mathematics, the students should be encouraged to select models such as thinking diagrams, tree diagrams, Ven diagrams, tables, charts, memorization, summaries, systematization of mathematical knowledge.

The students can search for mathematical solutions through the transformation of the representations. In particular, when the students must "translate" from a practical situation to a mathematical position by using the mathematical models (in the form of symbols, icons, etc....) and vice versa to solve the mathematical problems in a practical context.

**Example 5.** (Grade 7) There are two residential areas on the same side and not far from a highway. Find a location on the roadside to construct a medical station. This medical station is evenly spaced between two residential areas (a straight road that connects two residential areas is not perpendicular to the highway).

Here, the students must think, use drawings, symbols, icons to real model situations; use knowledge, mathematical skills to find out, argue, prove; use the mathematical language to present the solution to the problem and switch to the natural language the actual situation. It is specified as follows:

The students use two distinct points, A and B (representing two residential areas) and a straight-lined (representing highway) (Figure 2.a). Let C be the intersection of the midperpendicular AB (the road connecting the two residential areas) with the straight-lined. We have  $CA = CB$  (property of midperpendicular). So: C is the point to find (Figure 2.b).

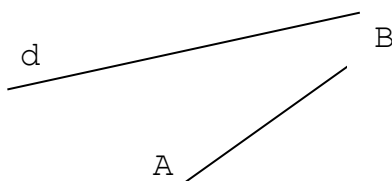


Figure 2.a

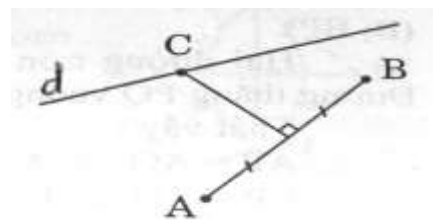


Figure 2.b

**Figure 2:** Representing two residential areas and highway (a); The intersection of the midperpendicular AB (b)

### CONCLUSION

The formation and organization of study cases for students to apply the mathematical representations are not only the premise to stimulate such activities but also contribute to clarify the orientation of teaching reformation under the development of mathematical competence for learners, increase the responsibility, activeness, and sense of initiative

of the learners during the establishment of mathematical understanding, creation of firm knowledge, formation, and development of the capacity of connecting the mathematics with the reality. In the context of reforming school mathematics, the research and construction of measures of improving the mathematical representation competence for students in teaching mathematics have been more and more



necessary, tending to the formation and development of competence and virtue of learners. Through the organization of such similar activities, the students obtain experiences on the transformation and "interpretation" between representations in every circumstance. The observation and comparison help students understand the similarity of the mathematical expressions when the exact relationship is represented. The transformability of mathematical models is flexibly formed during learning.

### RECOMMENDATIONS

Several recommendations can be drawn from this research results. It can be said that to form and develop the mathematical representation competence for students, teachers need to pay attention to the following matters during the process of teaching mathematics in junior high school:

- (1). Train the students to understand and use the representations correctly in a mathematics textbook.
- (2). Encourage the students to use and form various models during the process of teaching.
- (3). Encourage the students to present and assess the use of representations in solving mathematical problems.
- (4). Teachers always form and apply various terms for the same mathematical content.
- (5). Best exercises to train the students to use and develop the mathematical representations in many ways.

### LIMITATIONS

This research aims to build mathematical competencies for junior high school students by exploiting and using mathematical representations in the mathematical learning process. Future research may be available to elementary and high school students. Furthermore, future research should focus more on a broader sample of participants for a more general overview and perspective of participants from a more detailed interview.

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