



A Note on Dispersion Analysis in Two Phase Flows

Kanti Pandey¹, Preeti Verma², D.N. Tiwari³

^{1,2,3}Department of Mathematics & Astronomy, Lucknow University, Lucknow U.P. (India) 226007

ARTICLE INFO

Published Online:
07 January 2019

Corresponding Author:
Kanti Pandey

ABSTRACT

In present article a theoretical attempt is made to present the variation of attenuation by water droplets in air for various values of particle volume fraction and Prandtl number. It is concluded that as particle volume fraction increases dispersion has a decreasing trend while attenuation increases and attend maximum value and then decreases continuously. Similarly for fixed values of ξ , γ , ε_0 and for different value of Prandtl number dispersion decreases and reached minimum and then increases while attenuation increase and attend the maximum value and then decreases. In preparation of graphs MATLAB is used .

KEYWORDS: dispersion analysis , two – phase flows

INTRODUCTION

Sound propagation in a two phase medium has been studied from time to time. Sewell [15] investigated attenuation of sound caused by rigid particles suspended in a gas, assuming that the particle were immovable. As a consequence of this assumption, the predicted attenuation remains finite as particle size is reduced. As pointed out by Lamb [9] this result is not acceptable, because as particle radius is reduced, a point is reached where the particle moves with the gas and so causes little or no energy loss. Epstein [3] considered the attenuation of sound by spherical particles that were allowed to oscillate with amplitudes smaller than their radius. Epstein and Carhurt [4] made an important extension to this theory by including the dissipation due to irreversible heat transfer and compared it with exiting data taken by Knudsen et al [8]. First Zink and Delsasso [19] presented dispersion of sound by suspended particles and reported good agreement between the theory and their experimental result. Temkin and Dobbins [18] presented different approach to discuss theory for particulate attenuation and dispersion.

The study of wave propagation in a mixture of gas and dust particles has received great attention during the last several decades. There are many engineering applications for flow of a medium that consists of a suspension of powdered material or liquid droplets in a gas, such as flow in rockets, nuclear- reactors, fuel – sprays, air – pollution etc.. The mathematical analysis of such two phase flows is considerably more difficult than that of pure gas flows, and one of the usual simplifying assumptions is that the volume occupied by the particles can be neglected. In many important cases, the particles represent less than one half of

the mass of the gas particle mixture, and the density of the particle material is more than a thousand times larger than the gas density. Under such conditions, the particle volume fraction is of the order of 10^{-4} only and the assumption of a negligible particle volume is then well satisfied. At high gas densities (high pressure) or at high particle mass fractions, the particle volume fraction may become sufficiently large, so that it may be included into flow analysis without introducing significant errors. Since the particles may be considered as incompressible in comparison with the gas, the particle volume fraction enters into the basic flow equations as an additional variable.

The interesting properties of such two phase flows is that even equilibrium flows cannot be treated as perfect gas flows. Several authors [11- 14, 16, 17] have investigated the different aspects of non linear waves in such case of two phase flows. There are many engineering problem in which dilute phase of gas particles is a good approximation of actual conditions. In such cases due to the existence of solid particles in the gas, properties of mixture differ significantly from those of gas alone. Such types of studies have numerous applications in underground explosion [10].

BASIC EQUATION

The theory developed in this chapter is based on the following assumptions-

Two – phase flows model considered by Rudinger [14] is taken when particle volume fraction appears as an additional variable.

1. The gas is thermally and calorically perfect.
2. The density and specific heat of particle material are constant.

3. The particles are spherical, of uniform size, incompressible and uniformly distributed.
4. The size of the particle and their average separation are negligible as compared with significant dimension of the flow field.
5. Mass transfer between particles and gas is absent; i.e., evaporation, condensation, and chemical reactions are excluded.
6. The total heat transfer rate and drag force between particulate phase and gas phase is the sum of the effects due to each particle. This implies that the particle diameter is assumed to be much smaller than the distance between the particles.
7. The fluctuation of pressure, density and temperature produced by the acoustic wave are assumed to be so small as compared with their mean values that their squares or their cross products can be neglected.
8. The drag coefficient and Nusselt number are prescribed as functions of the particle Reynold number.
9. No external force (such as gravity or heat exchange) affect the mixture.
10. Viscosity and heat – conductivity of gas is neglected except for the interaction with the particles.
11. Typical relaxation process such as “EQUILIBRIUM - FLOW” for which relaxation is assumed to be infinitely fast.

Following the model developed by Rudinger[14] one dimensional wave propagation through a gas with spherical particles in suspension, is given by following equations

$$\bar{\rho}_{,t} + \bar{\rho} u_{,x} + u \bar{\rho}_{,x} + \left(\frac{\bar{\rho} \bar{\varepsilon}}{(1-\bar{\varepsilon})} \right) v_{,x} + \left(\frac{\bar{\rho} (v-u)}{(1-\bar{\varepsilon})} \right) \bar{\varepsilon}_{,x} = 0, \quad (2.1)$$

$$u_{,t} + uu_{,x} + \left(\frac{R}{(1-\bar{\varepsilon})} \right) \bar{T}_{,x} + \left(\frac{R \bar{T}}{(1-\bar{\varepsilon}) \bar{\rho}} \right) \bar{\rho}_{,x} + \left(\bar{\varepsilon} \frac{\bar{\rho}_p}{(1-\bar{\varepsilon})^2 \bar{\rho}} \right) \left(\frac{u-v}{\tau_v} \right) = 0, \quad (2.2)$$

$$\left. \begin{aligned} & \bar{T}_{,t} + \bar{T}(\gamma-1) u_{,x} + \frac{\bar{T}}{\bar{\rho}} \left(\frac{u(\bar{\varepsilon}-\gamma)}{(1-\bar{\varepsilon})} + u\gamma + \frac{\bar{\varepsilon} v(\gamma-1)}{(1-\bar{\varepsilon})} \right) \bar{\rho}_{,x} + \\ & \left(\frac{u(1-\bar{\varepsilon}\gamma)}{(1-\bar{\varepsilon})} + \frac{\bar{\varepsilon} v(\gamma-1)}{(1-\bar{\varepsilon})} \right) \bar{T}_{,x} + \bar{T} \bar{\varepsilon} \frac{(\gamma-1)}{(1-\bar{\varepsilon})} v_{,x} + \\ & \frac{\bar{T}}{(1-\bar{\varepsilon})} \{u+v(\gamma-1)-u\gamma\} \bar{\varepsilon}_{,x} + \left(\frac{\bar{\rho}_p C_m (\bar{T}-\bar{T}_p)}{\bar{\rho} \tau_T C_V (1-\bar{\varepsilon})} \right) - \\ & \left(\frac{\bar{\rho}_p (u-v)^2}{\bar{\rho} \tau_v C_V (1-\bar{\varepsilon})^2} \right) = 0, \end{aligned} \right\} \quad (2.3)$$

$$\bar{\varepsilon}_{,t} + v \bar{\varepsilon}_{,x} + \bar{\varepsilon} v_{,x} = 0, \quad (2.4)$$

$$v_{,t} + v v_{,x} = \frac{1}{\tau_v} \left(\frac{u-v}{(1-\bar{\varepsilon})} \right), \quad (2.5)$$

$$\bar{T}_{p,t} + v \bar{T}_{p,x} = \frac{1}{\tau_T} (\bar{T} - \bar{T}_p), \quad (2.6)$$

where u, ρ, \bar{T} are gas velocity, density, temperature. v, ρ_p, \bar{T}_p are particle velocity, density and temperature. C_m and C_V are specific heat of particle material and of gas at constant velocity respectively, $\bar{\varepsilon}$ is particle volume fraction and τ_T are dynamic relaxation time and thermal relaxation time of the p τ_v article respectively and is given by following relation,

$$\tau_T = \frac{3}{2} P_r \xi \tau_v, \quad (2.7)$$

where P_r is Prandtl number and $\xi = \frac{C_m}{C_p}$, C_p being specific heat of gas at constant pressure. Linearising equations

from (2.1) to (2.6) by putting

$\bar{T} = T_0 + T, \bar{p} = p_0 + p, \bar{\varepsilon} = \varepsilon_0 + \varepsilon, \bar{\rho} = \rho_0 + \rho, \bar{T}_p = T_0 + T_p, \bar{\rho}_p = \rho_0 + \rho_p$, we have following equations,

$$\rho_{,t} + \rho_0 u_{,x} + \rho_0 \frac{\varepsilon_0}{(1-\varepsilon_0)} v_{,x} = 0, \quad (2.8)$$

$$u_{,t} + \frac{R}{(1-\varepsilon_0)} T_{,x} + \frac{RT_0}{(1-\varepsilon_0)} \rho_{,x} + \frac{\varepsilon_0 \rho_{p0}}{(1-\varepsilon_0)^2 \rho_0} \frac{(u-v)}{\tau_v} \quad (2.9)$$

$$T_{,t} + T_0(\gamma-1)u_{,x} + \frac{(\gamma-1)\varepsilon_0 T_0}{(1-\varepsilon_0)} v_{,x} + \frac{\varepsilon_0 \rho_{p0}}{(1-\varepsilon_0)\rho_0} \frac{C_m}{C_v} \frac{(T-T_p)}{\tau_T} = 0, \quad (2.10)$$

$$\varepsilon_t + \varepsilon_0 v_{,x} = 0, \quad (2.11)$$

$$v_{,t} = \frac{1}{\tau_v} \frac{(u-v)}{(1-\varepsilon_0)}, \quad (2.12)$$

$$T_{p,t} = \frac{(T-T_p)}{\tau_T}. \quad (2.13)$$

METHOD OF SOLUTION

As we are interested in finding the general dispersion relation for periodic disturbance, assuming that T, u, ρ, ρ_p, v and T_p depend on x and t through a factor $e^{i(Kx-\omega t)}$ from equations (2.12) and (2.13), we have

$$v = \frac{u}{\{1 - i(1-\varepsilon_0)\omega\tau_v\}}, \quad (3.1)$$

$$T_p = \frac{T}{1 - i\omega\tau_T}. \quad (3.2)$$

where $K = k_1 + ik_2$ and ω is circular acoustic frequency. Substituting v and T_p from equation (3.1) and (3.2) in equation (2.8) to (2.11) and neglecting square and higher powers of ε_0 , after certain manipulation, they reduces into following set of equations,

$$-\omega\rho + \rho_0 K \left\{ 1 + \frac{\varepsilon_0}{\{1 - i(1-\varepsilon_0)\omega\tau_v\}} \right\} u = 0, \quad (3.3)$$

$$\frac{KR}{(1-\varepsilon_0)} T + \frac{KRT_0\rho}{\rho_0(1-\varepsilon_0)} - \left[1 + \frac{\eta}{\{1 - i(1-\varepsilon_0)\omega\tau_v\}} \right] \omega u = 0, \quad (3.4)$$

$$KT_0(\gamma-1)u \left\{ 1 + \frac{\varepsilon_0}{\{1 - i(1-\varepsilon_0)\omega\tau_v\}} \right\} - \omega T \left\{ 1 + \frac{\eta\xi}{1 - i\omega\tau_T} \right\} = 0, \quad (3.5)$$

$$-\omega\varepsilon + \varepsilon_0 K \frac{u}{\{1 - i(1-\varepsilon_0)\omega\tau_v\}} = 0. \quad (3.6)$$

The above equations can be written in the following matrix form,

$$\begin{pmatrix} 0 & -\omega & \left\{1 + \frac{\varepsilon_0}{\{1-i(1-\varepsilon_0)\omega\tau_v}\}}\right\} \rho_0 K & 0 \\ \frac{KR}{(1-\varepsilon_0)} & \frac{KR T_0}{\rho_0(1-\varepsilon_0)} & -\left(1 + \frac{\eta}{\{1-i(1-\varepsilon_0)\omega\tau_v}\}}\right) \omega & 0 \\ -\omega \left\{1 + \frac{\eta\gamma\xi}{1-i\omega\tau_T}\right\} & 0 & K T_0 (\gamma-1) \left\{1 + \frac{\varepsilon_0}{\{1-i(1-\varepsilon_0)\omega\tau_v}\}}\right\} & 0 \\ 0 & 0 & \frac{\varepsilon_0 K}{\{1-i(1-\varepsilon_0)\omega\tau_v}\}} & -\omega \end{pmatrix} \begin{pmatrix} T \\ \rho \\ u \\ \varepsilon \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad (3.7)$$

and have a solution if only if following determinant vanishes which means

$$\begin{vmatrix} 0 & -\omega & \left\{1 + \frac{\varepsilon_0}{\{1-i(1-\varepsilon_0)\omega\tau_v}\}}\right\} \rho_0 K & 0 \\ \frac{KR}{(1-\varepsilon_0)} & \frac{KR T_0}{\rho_0(1-\varepsilon_0)} & -\left(1 + \frac{\eta}{\{1-i(1-\varepsilon_0)\omega\tau_v}\}}\right) \omega & 0 \\ -\omega \left\{1 + \frac{\eta\gamma\xi}{1-i\omega\tau_T}\right\} & 0 & K T_0 (\gamma-1) \left\{1 + \frac{\varepsilon_0}{\{1-i(1-\varepsilon_0)\omega\tau_v}\}}\right\} & 0 \\ 0 & 0 & \frac{\varepsilon_0 K}{\{1-i(1-\varepsilon_0)\omega\tau_v}\}} & -\omega \end{vmatrix} = 0 \quad (3.8)$$

Expanding above determinant we see that K should satisfy the following equation

$$\frac{K^2 a_0^2}{\omega^2(1-\varepsilon_0)} \left[\left(1 + \frac{\eta\xi}{1-i\omega\tau_T}\right) \left\{1 + \frac{\varepsilon_0}{\{1-i(1-\varepsilon_0)\omega\tau_v}\}}\right\} = \left\{1 + \frac{\eta}{\{1-i(1-\varepsilon_0)\omega\tau_v}\}}\right\} \left\{1 + \frac{\eta\xi\gamma}{1-i\omega\tau_T}\right\} \right], \quad (3.9)$$

which is required dispersion relation, where, $a_0^2 = \frac{\gamma P_0}{\rho_0}$.

Equating real and imaginary part of above equation we have following relations,

$$\begin{aligned} & \frac{(k_1^2 - k_2^2) a_0^2}{\omega^2(1-\varepsilon_0)} \left[\left(1 + \frac{\varepsilon_0}{P}\right)^2 + \left(\frac{\varepsilon_0 \omega \tau_v}{P}\right)^2 \right] \left[\left(1 + \frac{\eta\xi}{Q}\right)^2 + \left(\frac{\omega \tau_T \eta \xi}{Q}\right)^2 \right] = \\ & \left(1 + \frac{\eta}{P}\right) \left(1 + \frac{\varepsilon_0}{P}\right) \left[\left(1 + \frac{\eta\xi\gamma}{Q}\right) \left(1 + \frac{\eta\xi}{Q}\right) + \gamma \left(\frac{\eta\xi\omega\tau_T}{Q}\right)^2 \right] - (1-\varepsilon_0) \left(1 + \frac{\varepsilon_0}{P}\right) \frac{(\gamma-1)\eta^2\omega\tau_v\omega\tau_T}{PQ} + \\ & \varepsilon_0 \left[\left(1 + \frac{\eta}{P}\right) \frac{(\gamma-1)\eta\xi\omega\tau_v\omega\tau_T}{PQ} + (1-\varepsilon_0) \left[\frac{(\gamma-1)\eta^2\xi\gamma\omega\tau_v\omega\tau_T}{PQ} + \left(1 + \frac{\eta\xi\gamma}{Q}\right) \left(1 + \frac{\eta\xi}{Q}\right) \eta \left(\frac{\omega\tau_v}{P}\right)^2 \right] \right] \end{aligned} \quad (3.10)$$

and

$$\left. \begin{aligned} & \frac{2k_1 k_1 a_0^2}{\omega^2(1-\varepsilon_0)} \left[\left(1 + \frac{\eta\xi}{Q}\right)^2 + \left(\frac{\eta\xi\omega\tau_T}{Q}\right)^2 \right] \left[\left(1 + \frac{\varepsilon_0}{P}\right)^2 + \left(\frac{\varepsilon_0\omega\tau_v}{P}\right)^2 \right] = \\ & \frac{(1-\varepsilon_0)\eta\omega\tau_v}{P} \left(1 + \frac{\varepsilon_0}{P}\right) \left[\left(1 + \frac{\eta\xi}{Q}\right) \left(1 + \frac{\eta\xi\gamma}{Q}\right) + \gamma \left(\frac{\eta\xi\omega\tau_T}{Q}\right)^2 \right] + \\ & \left(1 + \frac{\varepsilon_0}{P}\right) \left(1 + \frac{\eta}{P}\right) (\gamma-1) \frac{\eta\xi\omega\tau_T}{Q} - \\ & \frac{\varepsilon_0\omega\tau_v}{P} \left[\left(1 + \frac{\eta}{P}\right) \left\{ \left(1 + \frac{\eta\xi}{Q}\right) \left(1 + \frac{\eta\xi\gamma}{Q}\right) + \gamma \left(\frac{\eta\xi\omega\tau_T}{Q}\right)^2 \right\} + (1-\varepsilon_0)(\gamma-1) \frac{\eta^2\xi\omega\tau_T\omega\tau_v}{PQ} \right] \end{aligned} \right\}, \quad (3.11)$$

where

$$p = \left\{ 1 + (1 - \varepsilon_0)^2 \omega^2 \tau_v^2 \right\}$$

$$Q = (1 + \omega^2 \tau_T^2)$$

Equation (3.10) and (3.11) can be solved for k_1 and k_2 which are connected with speed of sound a and attenuation coefficients α by following relation

$$a = \frac{\omega}{k_1}, \tag{3.12}$$

and

$$\alpha = 2k_2. \tag{3.13}$$

RESULT AND DISCUSSION

If mass concentration η^2 and particle volume fraction ε^2 be neglected as compared to unity, we have cases which includes fogs, clouds and a great number of artificially produced smokes, thus neglecting η^2 and ε^2 from equations (3.10) and (3.11), we have

$$\frac{(k_1^2 - k_2^2)a_0^2}{\omega^2} = (1 - \varepsilon_0 - \frac{3\varepsilon_0}{P}) + \eta \left[\left\{ \frac{\xi(\gamma - 1)}{Q} + \frac{1}{P} \right\} (1 - \varepsilon_0 - \frac{3\varepsilon_0}{P}) + \varepsilon_0 \left\{ \frac{\omega\tau_T\omega\tau_v\xi(\gamma - 1)}{PQ} + \left(\frac{\omega\tau_v}{P} \right)^2 \right\} \right] \tag{4.1}$$

$$\frac{2k_1k_2a_0^2}{\omega^2} = (1 - 2\varepsilon_0 - \frac{\varepsilon_0}{P}) \frac{\eta\omega\tau_v}{P} + (\gamma - 1)(1 - \varepsilon_0 - \frac{\varepsilon_0}{P}) \frac{\eta\xi\omega_T}{Q} - \frac{\varepsilon_0\omega\tau_v}{P} \left(1 + \frac{\eta\xi(\gamma - 1)}{Q} + \frac{\eta}{P} \right) \tag{4.2}$$

Using equations (3.12) and (3.13) from equation (4.1) and (4.2) we have following equations of attenuation and dispersion of sound by suspended particles.

$$F_1 = \frac{\frac{\alpha a_0}{\omega} + \frac{\varepsilon_0\omega\tau_v}{P}}{\eta} = \left[\frac{\omega\tau_v}{P} \left\{ (1 - 2\varepsilon_0 - \frac{\varepsilon_0}{P}) - \varepsilon_0 \left(\frac{\xi(\gamma - 1)}{Q} + \frac{1}{P} \right) \right\} \right] + (\gamma - 1)(1 - \varepsilon_0 - \frac{\varepsilon_0}{P}) \frac{\xi\omega_T}{Q} \tag{4.3}$$

$$F = \frac{\left(\frac{a_0}{a} \right)^2 - (1 - \varepsilon_0 - \frac{3\varepsilon_0}{P})}{\eta} = \left[\left\{ \frac{\xi(\gamma - 1)}{Q} + \frac{1}{P} \right\} (1 - \varepsilon_0 - \frac{3\varepsilon_0}{P}) + \varepsilon_0 \left\{ \frac{\omega\tau_T\omega\tau_v\xi(\gamma - 1)}{PQ} + \left(\frac{\omega\tau_v}{P} \right)^2 \right\} \right] \tag{4.4}$$

Putting $\varepsilon_0 = 0$ the result coincide with results obtained by Temkin and Dobbins [18] . Fig.1 represents the variation of dispersion by water droplets in air for various values of ε_0 . As ε_0 is increasing dispersion has a decreasing trend.

Fig.2 represents attenuation by water droplets for different value of ε_0 . As ε_0 is increasing attenuation has a increasing trend and reached maximum and then has decreasing tendency for fixed values of $\xi = 4.17, \gamma = 1.4, P_r = .71$.

Fig.3 represents attenuation for fixed values of $\varepsilon_0 = .75, \lambda = 1.4, \xi = .71$ and various values of prandtl number $P_r = .71, .81, .91$. As Prandtl number increasing attenuation increases and reached maximum and then has decreasing tendency for fixed values of $\xi = 4.17, \gamma = 1.4, \varepsilon_0 = .75$.

Fig.4 represents dispersion by water droplets for fixed values of $\xi = 4.17$, $\gamma = 1.4$, $\varepsilon_0 = .75$ and various values of Prandtl number, $P_r = .71, .81, .91$. As P_r increases dispersion decreases and reach to minimum and then increases and attend maximum.

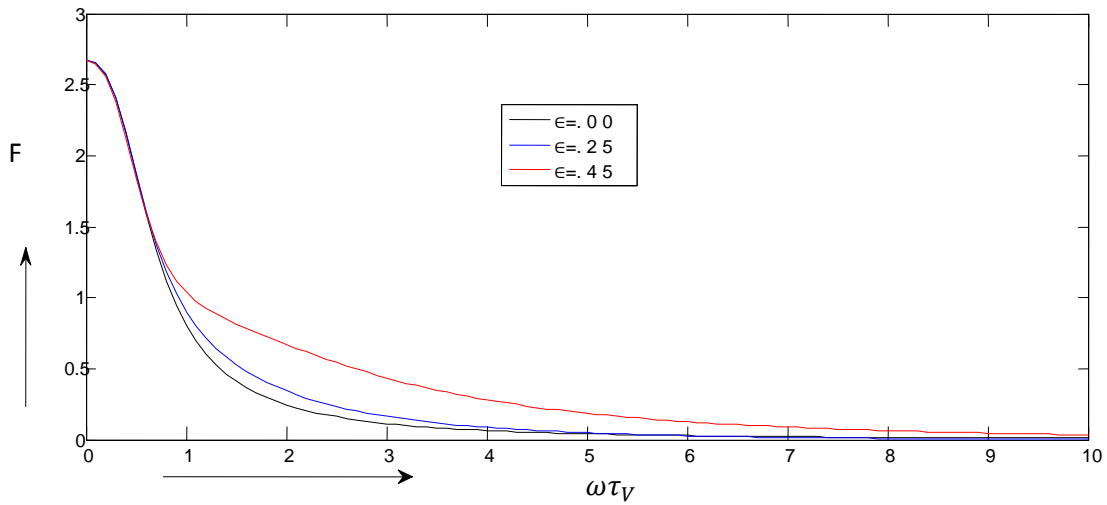


Fig.1: Dispersion by water droplets in air($P_r = .71, \gamma = 1.4, \xi = 4.17$)

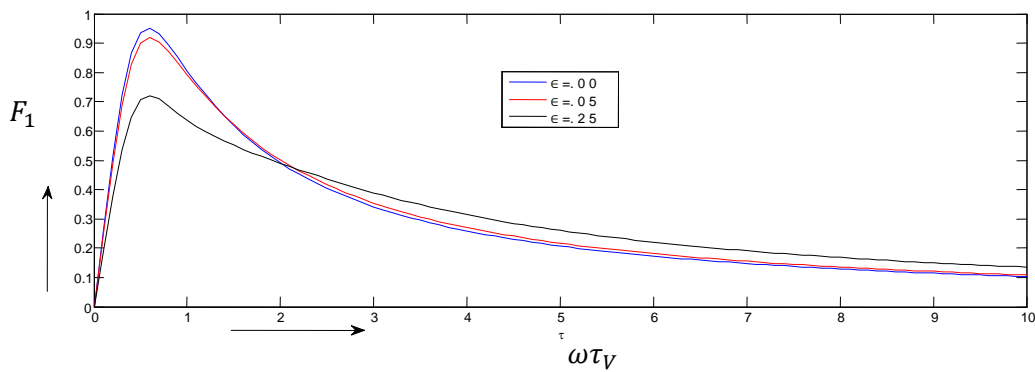


Fig.2: Attenuation by water droplets in air($P_r = .71, \gamma = 1.4, \xi = 4.17$)

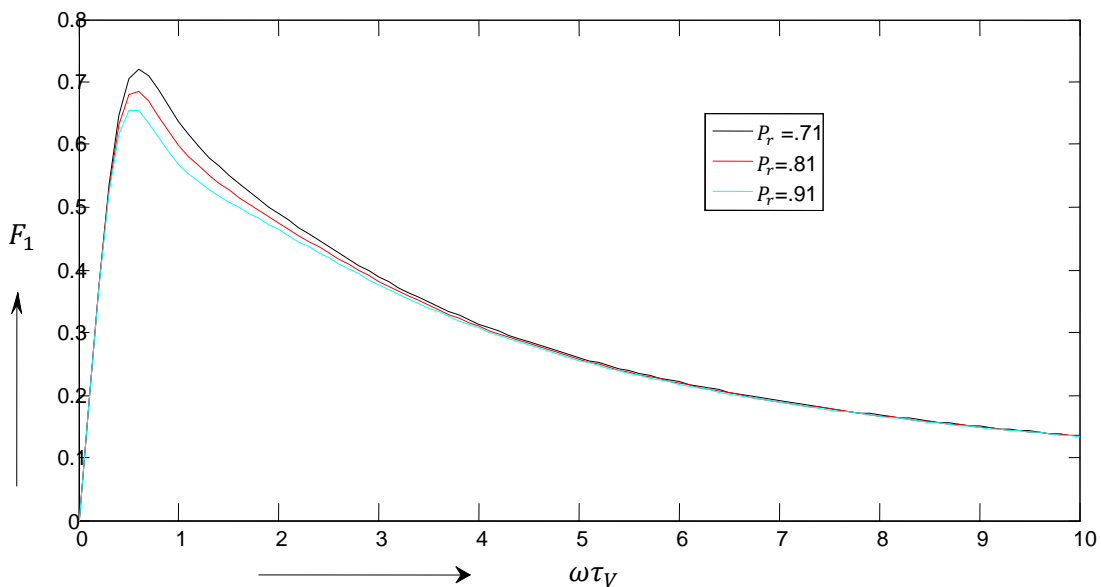


Fig.3: Attenuation by water droplets in air(for different value of Prandtl number.71, 0.81,.91) for fixed value of $\varepsilon_0 = .75, \lambda = 1.4, \xi = .71$.

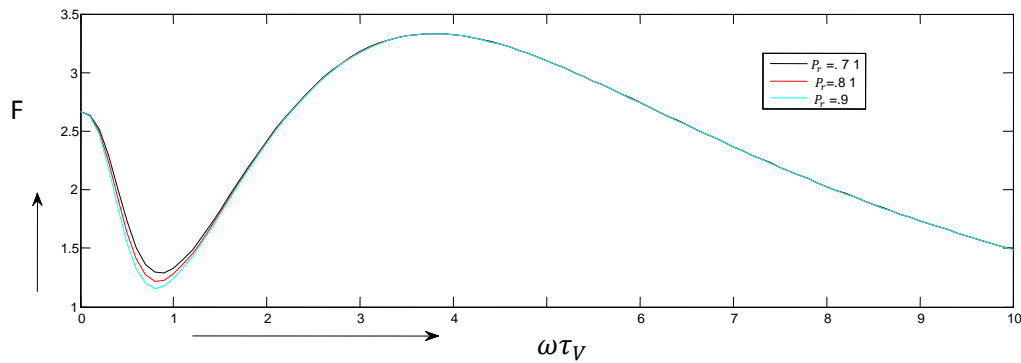


Fig.4: Dispersion by water droplets in air (for different value of Prandtl number 7.1, 0.81, .91) for fixed value of $\varepsilon_0 = .75, \lambda = 1.4, \xi = .71$.

Acknowledgement: Authors Kanti pandey and Preeti Verma are grateful to U.G.C. for providing financial assistance in preparation of this article.

REFERENCES

1. Chu, B. T. – June, -(1960) - Thermodynamics of a Dusty Gas and Its Application to some Aspects of Wave Propagation in the Gas- Brown Univ. Rept. No.DA-4761/1.
2. Chow, J. C. F. – (1964) - Attenuation of Acoustic Waves in Dilute Emulsions and Suspensions - J. Acoust. Soc.Am. - Vol. 36 - pp. 2395 - 2401.
3. Epstein, P.S. – (1941) – On the Absorption of Sound by Suspensions and Emulsions - in Contributions to Applied Mechanics - Theodore von Kármán Anniversary Volume (California Institute of Technology -Pasadena) – pp.162 – 188 .
4. Epstein, P.S. and Carhart, R. R. – (1953) – The Absorption of Sound by Suspensions and Emulsions in.I.Water Fog in Air - J. Acoust. Soc.Am. – Vol. 25 - pp. 553 – 565 .
5. Fuchs, N. A. – (1964) – The Mechanics of Aerosols, The Macmillan Co. - Inc.- New – York - pp. 84.
6. Herzfeld, K. F. and Litovitz, T. A. – (1959) – Absorption and Dispersion of Ultrasonic Waves - Academic Press Inc. - New – York.
7. Jena, J. & Sharma, V.D.- (1999) - Self Similar Shocks in a Dusty Gas- International J. of Non-linear Mechanics- Vol.34- pp. 313 – 327 .
8. Knudsen, V. O., Wilson, J. V. and Anderson, N. S. – (1948) - The Attenuation of Audible Sound in Fog and Smoke - J. Acoust. Soc.Am. –Vol. 20 - pp. 849 – 857 .
9. Lamb,H. – (1945) – Hydrodynamics (Dover Publications Inc.) - New - York - 6th Ed. -pp. 659 .
10. Lamb, F.K., Collen, B. W. and Sullivan, J. D.- (1992)- An approximate analytical model of shock waves from underground nuclear explosion- J. Geophys. Res. *B*₁ - Vol. 97- pp. 515 – 535 .
11. Mishra, R.S. and Srivastava, O.S.- (1965) - Three dimensional shock waves in a dusty medium- Tensor N.S.- Vol.16- no.2 - pp.133 – 142 .
12. Marble, F. E. – (1963) –Dynamics of a Gas Containing Small Solid Particles - Proc. AGARD Combustion Propulsion Colloq. - 5th pp.175 - 215.
13. Pai, S. I., Menon S. and Fan, Z.Q.- (1980)- Similarity solutions of a strong shock wave propagation in mixture of a gas and dusty particles – Int. J. of Eng. Sci. – vol.18- pp. 1365 – 1373 .
14. Rudinger, G. - (1964)-Some properties of shock relaxation in gas flows carrying small particles - Phys. of Fluid-Vol. 7- pp.658 – 663 .
15. Sewell, C.J.T.- (1910) - On the Extinction of Sound in a Viscous Atmosphere by Small Obstacles of Cylindrical and Spherical Form – Phil. Trans.- Roy. Soc. (London) -Vol. A210, pp. 239 – 270 .
16. Soo, S.L. - (1961) - Gas dynamic processes involving suspended solids- A.I.Ch. E.J. - Vol.7- no.3 - pp.384 – 391 .
17. Sharma, V. D. and Gupta, N. - (1992)- Propagation of rapid pulses through a two phase mixture of gas and dust particle - Int. J. Eng. Sci.- Vol. 30- pp. 263 – 272 .
18. Temkin, S. and Dobbins, R.A.- (1996) - Attenuation and dispersion of sound by particulate-relaxation process- J. Acoust. Soc. Am. - Vol. 40 – no.2- pp. 317 – 324 .
19. Zink, J. W. and Delsasso, L. P. – (1958) - Attenuation and Dispersion of Sound by Solid Particles Suspended in a Gas - J. Acoust. Soc. Am.- Vol. 30 - pp.765 – 771 .