RA Journal of Applied Research
||Volume||2||Issue||08||Pages-571-578||August-2016|| ISSN (e): 2394-6709
www.rajournals.in

Elegant labeling of some graphs and their line graphs

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***Abstract:** An elegant labeling f of graph G with ' q ' edges an injective function from the vertices of G to the set {0, 1, 2,------- q} such that when each edge (e = xy) is assigned the label { (f (x) + f (y)) mod(q + 1) } the resulting edge labels are distinct and non zero. In this paper it is shown to be certain families of line graphs of elegant graphs are elegant graphs.*

***Key words:**Path graph,P^2_n,Comb graph, H_{nn},B_{nn}.*

1.INTRODUCTION:

We consider all graphs are finite, simple and undirected. The graph G has 'V' vertices and 'E' edges. A graph labeling is an assignmentof integers to the vertices or edges or both subject to certain condition.We refer to survey on graph labeling by Gallian [5]. An elegant labelingwas introduced by Chang, Hsu and Rogers in 1981 [2] have established the elegantness of C_n and P_n. Balakrishnan ,Selvam and Yengnanaryan [9] have shown that the H_{nn},B_{nn} are elegant if n is even.Recently V. Laxmi, Alias Gomathi, N murugan and A.Nagarajan [10] have shown that the P^2_n ,P_nK_1, are elegant Graphs. The definition and other information's which are used for present investigation are given.

2.Definitions:

Definition 2.1: Elegant graph : An elegant labeling f of graph G with 'q' edges is an injective function from the vertices of G to the set {0, 1, 2,------- q} such that when each edge (e = xy) is assigned the label { (f (x) + f (y)) mod(q + 1) } the edge labels are distinct and non zero, the resulting graph is called an elegant graph.

Definition 2.2: $H_{n,n}$: The graph with vertex set V ($H_{n,n}$) ={v_1,v_2,v_3,----v_n; u_1,u_2,---u_n } and edge set E ($H_{n,n}$) = {v_iu_j;1≤ i≤ n,n-i+1≤j≤n}.[9]

Definition 2.3:P^2_n: The graph P^2_n is a graph with vertex set V (P^2_n) ={ u_i;1≤ i≤ n } and E(P^2_n) = {u_iu_{i+1} ;1≤ i≤ n-1} U {u_iu_{i+2} ;1≤ i≤ n-2}.[10]

Chacterstics of labeling:

- The vertex labeling L(G) chosen from the integers set{0,1,2,3,----q}
- Label the vertices in clockwise/anticlockwise/randomly in an increasing order.
- It should satisfy f = (xy) = { (f (x) + f (y)) mod(q + 1) }.
- The condition holds good for few L(G) and it can't be generalized.
- L(G) has many edges and enough vertices.

3.MAIN RESULTS:

Theorem 3.1: The line graph of (P^2_n- e) is an elegant graph if n ≡ 1(mod 2), n ≥ 3 and (e = n-4).

Proof: Let G =(P^2_n)be a graph with ' p' vertices and 'q' edges and it's line graph [L (P^2_n)] {u_1,u_2,u_3,-------u_n; v_1, v_2, v_3, v_4------------v_n} be the vertices and the edge set be{e_1, e_2,e_3, -----e_n}.

We define the labeling functionf :V(G) → {0,1,2, 3,---------------.q} as follows.

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$f(u_1) = 0$

$f(u_2) = 1$

$f(u_{2i+1}) = 5i \quad \text{for } 1 \le i \le \left[\frac{q-p}{2} \right]$

$f(u_{2i+2}) = 5i + 1 \quad \text{for } 1 \le i \le \left[\frac{q-p}{2} \right]$

$f(v_1) = 2$

$f(v_{2i}) = 4i + j \quad \text{for } 1 \le i \le \left[\frac{q-p}{2} \right] \text{ and } 0 \le j \le \left[\frac{q-p}{2} - 1 \right]$

$f(v_{2i+1}) = 7i - j \quad \text{for } 1 \le i \le \left[\frac{q-p}{2} \right] \text{ and } j = 0, 2, 4, \text{--------} (n-5)$

The above defined labeling pattern easy to verify that all the vertex labels are different values and we get edge label in the form{ 1, 2, 3,------- q} non – zerointegersand no edge label is repeated.Hence L(P^2_n - e) is a elegantlabeling graph.

Example.L(P^2_5) is a elegant labeling graph as shown in fig 1 **n=5 e = n - 4.**

e=1 (one edge deleted)

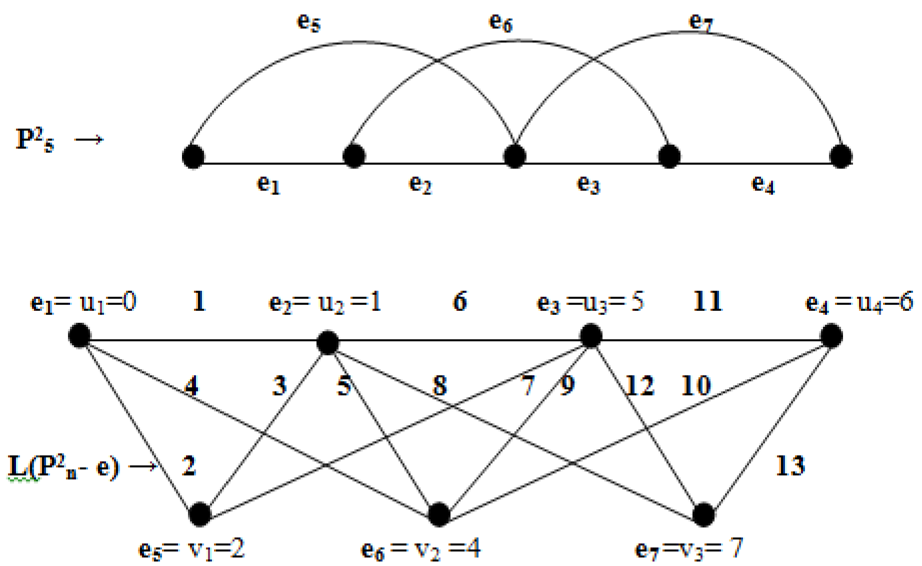

fig 1.

Theorem 3.2: The line graph of (P^2_n- e) is an elegant graph if $n \equiv 0 (\bmod 2)$,$n \ge 4$and (e = n-4).

Proof: Let G =(P^2_n)be a graph with ' p' vertices and 'q' edges and it's line graph [L (P^2_n)] { v_1, v_2, v_3, v_4 ------------ v_n;u_1,u_2,u_3,-------u_n } be the vertices and edge set {e_1, e_2, e_3, ----------e_n }.

We define the labeling function f : V (G) → { 0, 1, 2, 3,---------------.q} as follows.

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$$f(v_{1+2i}) = 5i \quad \text{for } 0 \le i \le \left(\frac{n}{2} - 1\right)$$

$$f(v_{2+2i}) = 3+5i \quad \text{for } 0 \le i \le \left(\frac{n}{2} - 2\right)$$

$$f(u_{1+2i}) = 5i+1 \quad \text{for } 0 \le i \le \left(\frac{n}{2} - 2\right)$$

$$f(u_{2+2i}) = 2+5i \quad \text{for } 0 \le i \le \left(\frac{n}{2} - 2\right)$$

The above defined labeling pattern easy to verify that all the vertex labels are different values and we get edge label in the form{ 1, 2, 3,------- q} non – zero integers and no edge label is repeated. Hence L(P^2_n- e) is a elegant labeling graph.

Example.L(P^2_6) is a elegant labeling graph as shown in fig 2**n=6 e = n - 4 .**

e=2 (two edge deleted)

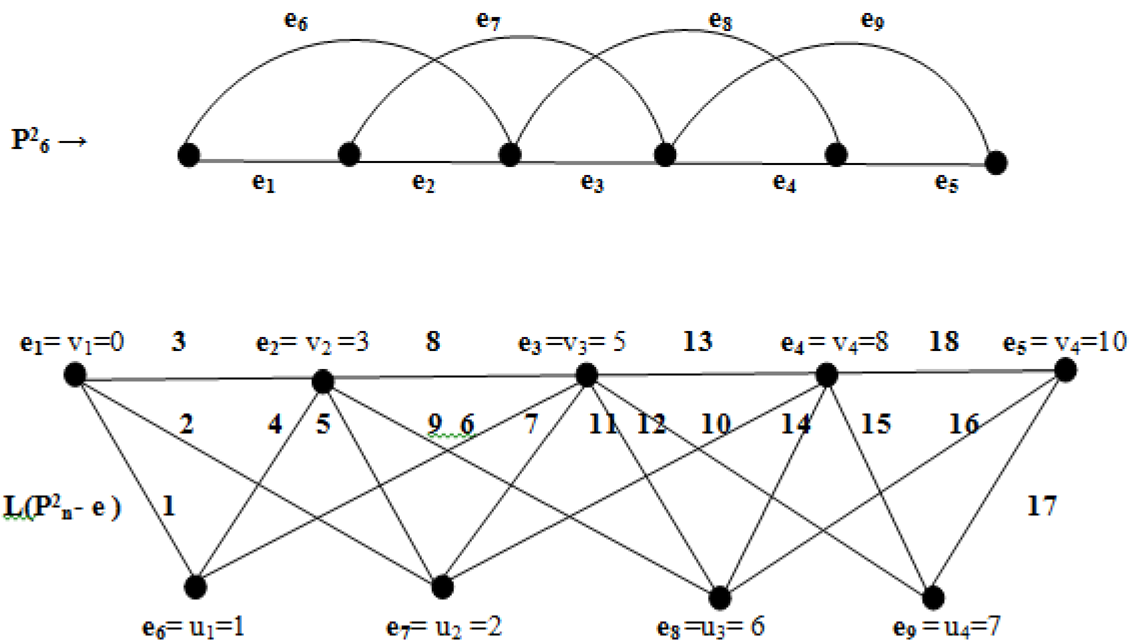

fig 2.

Theorem 3.3: The line graph of ($P_n \odot K_1$) is an elegant graph if $n \le 5$

Proof: Let G =($P_n K_1$) be a comb graph with 'V' vertices and 'E' edges and it's line graph [L($P_n \odot K_1$)]{ x_0, x_1, x_2, x_3 ------------ x_n; y_1, y_2, y_3,-------y_n } be the vertices and edge set {e_1, e_2, e_3, ----------e_n }.We define the labeling function f : V (G) → { 0, 1, 2, 3,---------------.q} as follows.**Case 1:n=2**

$f(x_1) = 0$

$f(y_1) = 1$

$f(y_2) = 2$

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Case 2 :n=3

$f(x_i) = i$ for $0 \leq i \leq n - 2$

$f(y_i) = i+1$ for $1 \leq i \leq n$

Case 3:n=4

$f(x_{1+2i}) = 5i+1$ for $i=0,1$

$f(x_2) = 2$

$f(y_{1+2i}) = 5i$ for $i=0,1$

$f(y_{2+2i}) = 3i+4$ for $i=0,1$

Case 4:n=5

$f(x_i) = i$ for $0 \leq i \leq n - 2$

$$f(y_{1+2i}) = (n-1) + i(i+1) + i \quad \text{for } 0 \leq i \leq \left(\frac{n-1}{2} \right)$$

$f(y_{2i}) = (n-1) + 2i$ for $i=1,2$

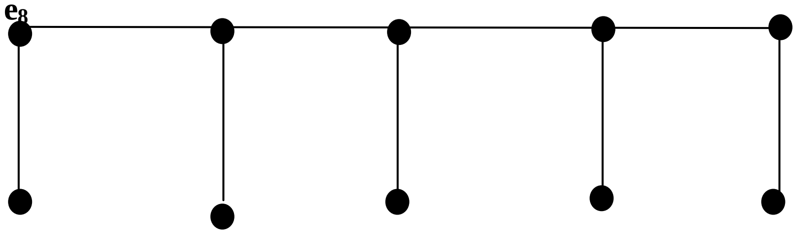
The vertex labeling pattern defined above covers all the vertices with different values and we get edge label in the form{ 1, 2, 3,------- q} non – zero +ve integers and no edge label repeated.Hence L(P_nK_1) is ● elegant labeling graph.

Example.L (P_5K_1●) is a elegant labeling graph as shown in fig 3

e_2 $e_4$$e_6$ **e_8**

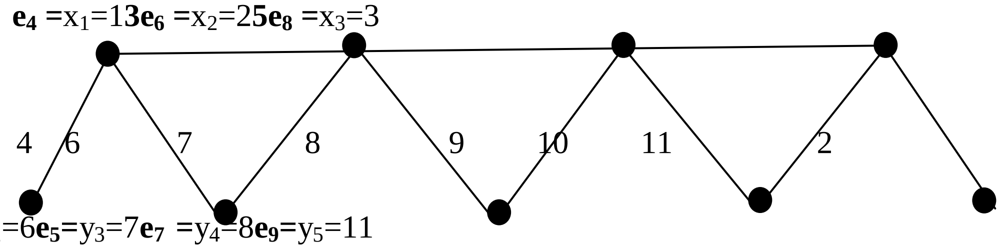
P_5K_1● →

$e_1$$e_3$ $e_5$$e_7$$e_9$



e_2 $=x_0=01$ **e_4** $=x_1=1$**3e_6** $=x_2=2$**5e_8** $=x_3=3$

L (P_5●K_1) →



e_1 $=y_1=4$**e_3**$=y_2=6$**e_5**$=y_3=7$**e_7** $=y_4=8$**e_9**$=y_5=11$

fig 3

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Theorem 3.4: The line graph of ($H_{n\,n}$) is a elegant graph if $n \leq 3$.

Proof: Let G be a($H_{n\,n}$) graph with 'p' vertices and 'q' edges and it's line graph L($H_{n\,n}$) { v_1, v_2, v_3, v_4------ v_n}vertices and edge set {e_1, e_2, ------ e_n}. We define the labeling function f : V (G) →{ 0,1, 2---------n – 1 } as follows

Case 1: n=2

$f (v_1) = 0$

$f (v_2) = 1$

$f (v_3) = 2$

Case 2:n=3

$f (v_1) = 6$

$f (v_2) = 0$

$f (v_{2+i}) = i$ for i=1,2,3,4

such that the vertex labels are different numbers and we get edge label in the form{ 1, 2, 3,------- q} non – zero integers and no edge label is repeated.Hence L(H_{nn}) is an elegant labeling graph.

Example.L(H_{33}) is a elegant labeling graph as shown in fig 4

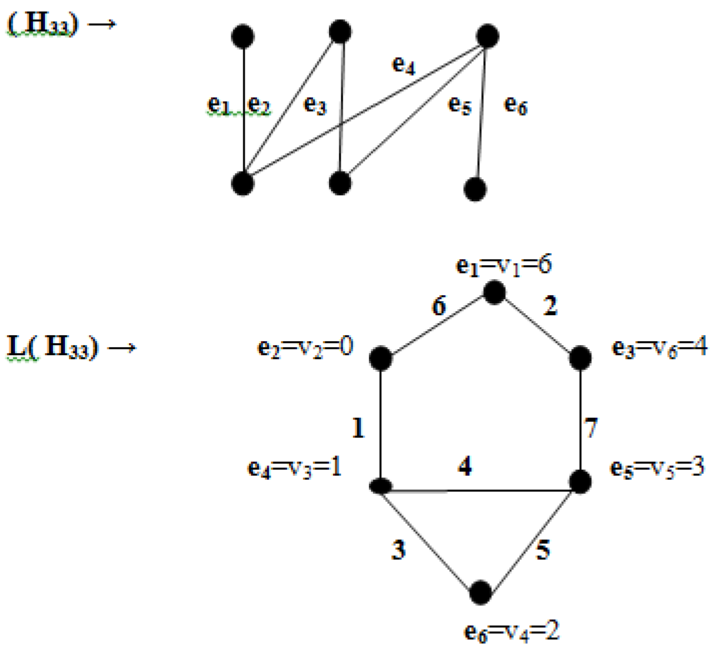

fig4

Theorem 3.5: The line graph of P_n is an elegant graph if $n \equiv 0(\bmod 2)$.

Proof: Let P_n be a path graph with a vertex set as{ v_1, v_2, v_3, v_4------ v_n},{ e_1, e_2, ------ e_{n-1}} be the edge and it's line graph L(P_n).has 'P'vertices and ' P -1'edges.

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We define the labeling function $f : V(G) \rightarrow \{0, 1, 2 \text{---------} q\}$ as follows

$$f(u_i) = \left(\frac{p-1}{2} + i \right) \bmod (q+1) \quad \text{for } 1 \le i \le P$$

The vertex labeling pattern defined above covers all the vertices with different numbers.Label Verticesfrom right to left side of the path and weget edge label in the form of { 1, 2, 3,------- q} , non – zero integers and no edge label is repeated.Hence $L(P_n)$ is a elegant labeling graph.

Example . $L(P_6)$ is a elegant labeling graph as shown in fig 5

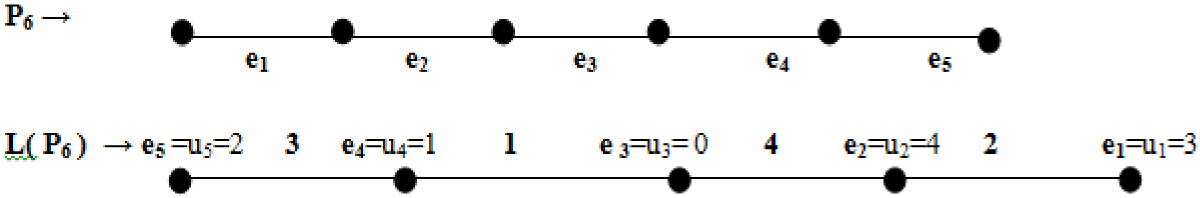

fig 5

Theorem 3.6: The line graph of ($\mathbf{B}_{nn}$) is a elegant graph if n=2

Proof: Let G be a bistargraph with v_1, v_2, v_3, v_4 ------ v_n be the vertices and edges e_1, e_2 ,---------- $e_{n\,1}$ it's line graph $L(B_{nn})$.has 'V'vertices and 'E'edges.We define the labeling function $f : V(G) \rightarrow \{0, 1, 2 \text{---------} q\}$ as follows

$f(x_i) = 0$

$f(v_{i+1}) = 3i+1$ for i=0,1

$f(u_{i+1}) = 3i+3$ for i=0,1

The above defined labeling pattern we can label the vertices with different values and weget edge label in the form{ 1, 2, 3,------------ q} non – zero integers and no edge label is repeated.Hence $L(B_{nn})$ is a elegant labeling graph.

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Example. L(B_{22}) is a elegant labeling graph as shown in fig 6

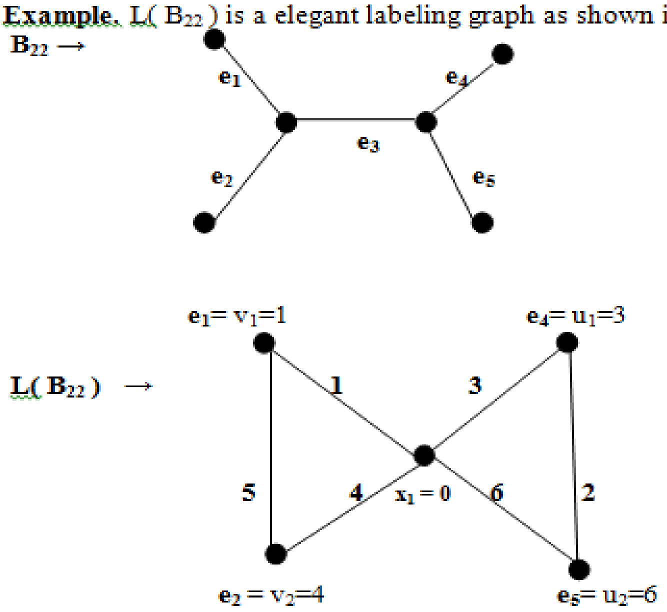

fig 6

CONCLUSION:

In this paper we have shown that line graph of P^2_n , Comb graph, Path graph, H_{nn}, B_{nn} are elegant graphs it can also verified for some graph**s.**

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