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Sequential Forest Harvesting In A Stochastic Environment

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ABSTRACT: The present paper extends Faustman model to incorporate uncertainty of forest growth and develop sequential forest harvesting strategies, by making use of the optimal stopping theory. More precisely, we formulate what follows the geometric Brownian motion to be the increment of the forest stock, not the forest stock itself, assuming the drift parameter to be negative. It is revealed that frequency of harvesting should decrease if uncertainty of forest growth increases or harvesting becomes more costly.

INTRODUCTION

When should we harvest the forest? This is the issue that has been investigated extensively since Faustman (1849) founded the basis of researches on forest harvesting. Kilkki and Väisänen (1969) and Näslund (1969) introduced thinning to the Faustmann model, while Chang (1982, 1983) extended landowner choices to include management effort to improve the growth conditions of the stands. Heaps and Neher (1979) and Heaps (1984) investigate the consequences of various restrictions imposed on harvesting capacity, while McConnell et al. (1983) and Newman et al. (1985) examine the case where timber price and regeneration costs change over time.

The present paper attempts to push forward these studies by incorporating uncertainty. More precisely, the present study develops the optimal strategy of a land owner who periodically plants and harvests trees in a stochastic environment, by constructing a stochastic dynamic model based on the optimal stopping theory. The optimal stopping theory, which emphasizes the importance of flexibility, is a theory that has been used to develop strategies in various stochastically fluctuating economies. For example, Dixit (1989) examines the timing of entering foreign market, while Farzin, Huisman and Kort (1988) investigate the timing of IT investment. Bentolila and Bertola (1990) consider the timing of employment/lay-off, just to mention a few. As for its application to the forest harvesting, among the first are Clarke and Reed (1989), followed by Alvarez and Koskela (2003) who focus on uncertainty of interest rate, Insley and Rollins (2005) who lay out a two-factor model with linear growth and mean-reverting prices and so on.

Although the above articles successfully develop optimal stochastic harvesting strategies, the stochastic process they use is the geometric Brownian motion that means forests grow without bound over time, which is not proper description of the reality since it is often the case that forests increase at a decreasing rate although at first increase at an increasing rate. Thus, in the present paper, we formulate what follows the geometric Brownian motion to be the *increment* of the forest stock, not the forest stock itself, assuming the drift parameter of the geometric Brownian motion to be negative, in order to fill the gap between the precedence researches that use the geometric Brownian motion and the stylized fact that forests grow at a decreasing rate.

Besides, main focus of the precedence researches is on one shot move of a land owner, which is not an adequate extension of Faustman (1849) who pays attention to ongoing harvesting strategies. Having this in mind, the present paper attempts to develop sequential forest harvesting strategies by utilizing Fujita (2007, 2008), which determine sequential product introducing strategies for retailers. Chang (2005), the most related work, also pays attention to the ongoing strategies, but differs from the present paper, since Chang (2005) is based on the geometric Brownian motion.

Structure of this paper is as follows. Section 2 lays out a basic model and Section 3 describes the objective function of a land owner in a stochastic environment. Based on this formulation, Section 4 develops the optimal stochastic harvesting strategy of the land owner and examines its properties. Concluding remarks are made in Section 5.

BASIC MODEL

Let us consider a land owner who begins with bare land and periodically plant new trees and harvest them in a stochastic environment where time passes continuously and importance of the future diminishes with time, which we capture by the discount rate ρ.

For the simplicity of analysis, let us assume the amount of trees that are planted/harvested in each time to be unity, and define here the tree planted for the first time as the first generation's tree, and the tree for the second time as the second generation's tree and so on. Letting $S_i(t)$ and $R_i(t)$ denote the amount of the ith generation (*i.e.*, the forest stock

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of the ith generation) and its increment (*i.e.*, $\frac{dS_i(t)}{dt}$) at t, respectively, we assume that the increment of the amount of the tree at t, not the forest stock itself, follows the geometric Brownian motion of equation (1).

$$dR_i = -\mu R_i dt + \sigma R_i dz, \qquad (1)$$

with initial value R_0 for each generation's tree.

μ and σ are parameters of drift and volatility, with both μ and σ being positive constants. Larger μ means that the increment of the forest stock decreases more quickly; larger σ means that the growth as a whole is more uncertain. dz is Wiener process that expresses random movement, which has several real-world applications such as stock market fluctuations, exchange rate fluctuations and so on.

We specify the costs of planting and harvesting the ith tree as F and $\frac{1}{R_i*}$ respectively, where F is a positive constant. It is assumed here that the harvesting cost is larger if R_i* is smaller (*i.e.*, amount of the tree is larger). As for the unit price of timbers, on the other hand, we specify it to be unity in order to simplify the analysis.

If we let T_i be the interval between the plantation and harvesting of the ith tree with $T_0=0$, it follows that with reference to the ith tree, the land owner incurs the plantation cost at $\sum_{k=1}^{i-1} T_k$ and receives the revenues and incurs the harvesting cost at $\sum_{k=1}^{i} T_k$.

Since the present paper's model is stochastic, optimal timing of harvesting/planting is expressed by cut-off value of R_i rather than by the exact time point. Let R_i* denote the optimal cut-off value of R_i, where the land owner harvests the ith generation's tree and plants the $(i+1)th$ generation's tree.

DESCRIPTION OF THE OBJECTIVE FUNCTION

This section describes the objective function of the land owner.

First of all, let us derive the expected value of one unit of profit at T_1(*i.e.*, the expected value of $e^{-\rho T_1}$), when the land owner harvests the first generation's tree and plants the second generation's tree. If we let $G(R_0)$ denote this value, the general solution to $G(R_0)$ is expressed as

$$G(R_0) = A(R_0)^{\alpha_1} + B(R_0)^{\alpha_2}, \qquad (2)$$

where $\alpha_1<0$ and $\alpha_2>0$ are solution to the characteristic equation $\frac{1}{2}\sigma^2 x(x-1)-\mu x-\rho=0$. Since $G(R_0)$ satisfies $G(\infty)=0$ and $G(R_1*)=1$, it follows that $A=(\frac{1}{R_1*})^{\alpha_1}$ and $B=0$. Substituting these equations into (2) yields $G(R_0)=(\frac{R_0}{R_1*})^{\alpha_1}$. Letting α denote $-\alpha_1$, we obtain

$$G(R_0)=(\frac{R_1*}{R_0})^{\alpha}, \qquad (3)$$

where

$$\alpha = -\frac{\sigma^2+2\mu-\sqrt{(\sigma^2+2\mu)^2+8\rho\sigma^2}}{2\sigma^2}. \qquad (4)$$

Next, let us describe the amount of the first generation's tree when harvested at T_1, which is obtained as $R_0 - \frac{R_1*}{\mu}$ by integrating $R_1(t)$ over t from 0 to T_1. *i.e.*, by calculating $E[\int_0^{T_1} R_1 ds \mid R_1(T_1)=R_1*]$.

Noting that the land owner incurs the plantation cost at 0, while she/he receives the revenues and incurs the harvesting cost at T_1, we have the net present value of the profit of the first generation's tree as $(\frac{R*}{R_0})^{\alpha}(R_0 - \frac{R*}{\mu} - \frac{1}{R*})-F$.

If we assume symmetric solutions, *i.e.*, $R_1*= R_2*= \cdots = R*$, for the simplicity of analysis, sum of the net present value of each generation's tree that is harvested when the increment growth reaches $R*$, $V(R*)$, is described as

$$V(R*)=(\frac{R*}{R_0})^{\alpha}(R_0 - \frac{R*}{\mu} - \frac{1}{R*})-F$$

$$+(\frac{R*}{R_0})^{2\alpha}(R_0 - \frac{R*}{\mu} - \frac{1}{R*})-(\frac{R*}{R_0})^{\alpha}F + \cdots$$

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$$= \frac{1}{1-(\frac{R^*}{R_0})^{\alpha}}[(\frac{R^*}{R_0})^{\alpha}(R_0 - \frac{R^*}{\mu} - \frac{1}{R^*})\text{-}F]. \quad (5)$$

OPTIMAL STOCHASTIC ROTATING STRATEGY

Now we are ready to develop the land owner's optimal harvesting strategy, that is, the optimal value of R^*.

For this purpose, let us differentiate $V(R^*)$ with respect to R^* to obtain the following first order condition.

$$R_0 - \frac{R^*}{\mu} - \frac{1}{R^*} - F = \frac{1}{\alpha}(\frac{R^*}{\mu} - \frac{1}{R^*})\{1-(\frac{R^*}{R_0})^{\alpha}\}. \quad (6)$$

If we let $L(R^*)$ and $R(R^*)$ denote the left hand side and right hand side of (6) respectively, the optimal value of R^*, R^{*B}, is determined as the intersection of $L(R^*)$ and $R(R^*)$ as in Figure 1.

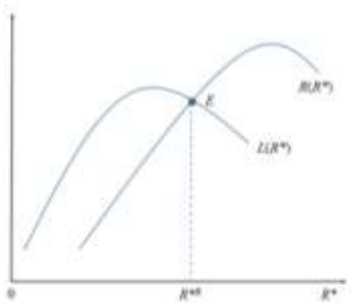

Figure1 determination of the optimal R^*

If we specify R_0=4, α=1.6, μ=1 and F=0.1 as case 1, for example, graph of $V(R^*)$ is drawn as in Figure 2.

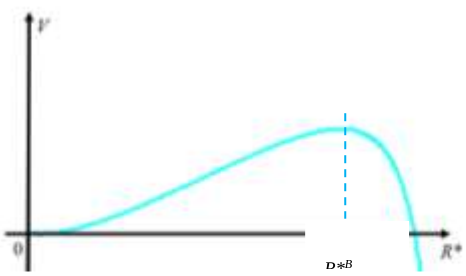

Figure2 graph of sum of the net present value

Now, let us focus on the uncertainty, which we capture by σ. If σ increases, $L(R^*)$ remains the same, while $R(R^*)$ shifts upward since $\frac{\partial \alpha}{\partial \sigma}<0$ from (4), with the optimal point decreasing from R^{*B} to $R^{*B'}$ as in Figure 3. Thus, we have $\frac{\partial R^{*B}}{\partial \sigma}<0$.

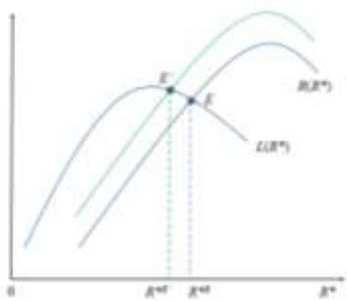

Figure3 effect of increase in uncertainty on the optimal R^*

If we specify R_0=4, α=0.8, μ=1, F=0.1 as case 2, in order to examine the effect of decrease in α, caused by increase in σ, on the net present value, we see that graph of $V(R^*)$ in case 2 is above that in case 1, with the optimal point decreasing from R^{*B} to $R^{*B'}$ as in Figure 4.

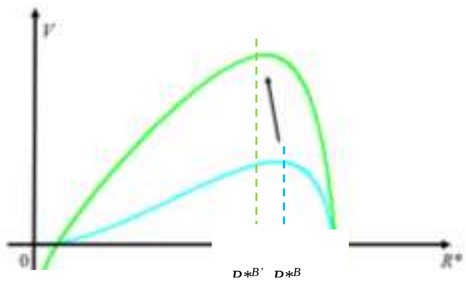

Figure4 effect of increase in uncertainty on V

Hence, the Proposition1 follows.

Proposition1: *If uncertainty of forest growth increases, the optimal cut-off value of the increment of forest stock decreases.*

This proposition indicates that if uncertainty of forest growth increases, frequency of harvesting decreases, which means the increase in the value of waiting.

We can also examine the effect of an increase in the costs, which we capture by an increase in F. In this case, $L(R^*)$ shifts downward, while $R(R^*)$ remains the same, with the optimal point decreasing from R^{*B} to $R^{*B'}$ as in Figure 3. Thus, we have $\frac{\partial R^{*B}}{\partial F}<0$.

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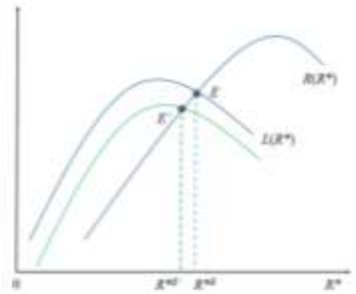

Figure5 effect of increase in cost on the optimal R^*

If we specify R_0=4, α=0.8, μ=1, F=1 as case 3, in order to examine the effect of an increase in F on the net present value, we see that graph of $V(R^*)$ in case 3 is below that in case 1 with the optimal point decreasing from R^{*B} to $R^{*B'}$ as in Figure 6.

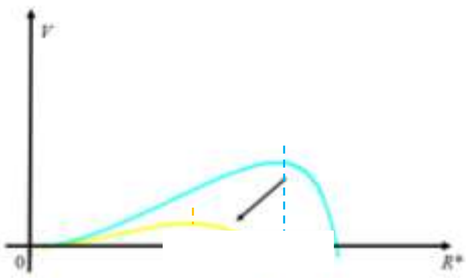

Figure6 effect of increase in cost increase on V

Proposition2: *If the cost of forest harvesting increases, the optimal cut-off value of the increment of forest stock decreases.*

This proposition implies that if the cost of forest harvesting increases, frequency of harvesting decreases.

CONCLUDING REMARKS

In this paper, we extended Faustman model to examine the properties of forest harvesting strategy in a stochastically fluctuating economy. Main result of this paper is: frequency of harvesting decreases if uncertainty of environment increases or cost of harvesting increases.

It is possible to extend the model to examine the effect of the change in the timber price. It is also necessary to relaxing the assumption that there is only one tree in each generation, to investigate the interaction between the trees. We take up such analysis next.

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