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# Migration In A Stochastic Environment:An Extension Of The Harris And Todaro Model Based On The Optimal Stopping Theory

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## ABSTRACT:

The present paper constructs an extended Harris-Todaro model that incorporates environmental stock, by making use of the optimal stopping theory, to examine the properties of migration in an economy where environment fluctuates stochastically. It is revealed that if uncertainty of environment in the rural area increases, inhabitants in the rural area should stay there longer, while, if the vulnerability of environment in the rural area increases, inhabitants in the rural area should leave there earlier, where uncertainty and vulnerability are respectively defined as the variance of stochastic fluctuation of environmental stock and the degree of sudden reduction of the environmental stock caused by some shock that happens from time to time.

## INTRODUCTION

Researches on migrations, which were founded by Todaro (1969), Harris and Todaro (1970) etc., have been making progress by incorporating the factors that were assumed away in the original Harris-Todaro model. Banergee (1983) lays out a model where difference in utilities between urban and rural areas triggers migration, while Jones and O'Neill (1995) focus on infrastructures in urban area. Nakagome (1989), Brueckner (1990), Brueckner and Zenou (1999), Brueckner and Kim (2001), on the other hand, shed light on land markets.

The present paper attempts to push forward these studies by incorporating the environmental stock, which is paid attention to by Fukuyama and Naito (2007). More precisely, by making use of the optimal stopping theory, we extend Fukuyama and Naito (2007) whose model is deterministic, to examine the effect of stochastic environmental fluctuation on migration.

The optimal stopping theory is a theory that has been used to develop strategies on timing in a stochastically fluctuating economy since McDonald and Siegel (1986) demonstrated the 'value of waiting.' Dixit (1989) examines the timing of entering foreign market, while Farzin, Huisman and Kort (1988) investigate the timing of IT investment. Bentolila and Bertola (1990) consider the timing of employment/lay-off. Leahy (1993), Caballero and Pindyck (1996) and Baldursson and Karatzas (1997), on the other hand, analyze the nature of stochastic fluctuating economy.

Although the above articles construct various sophisticated frameworks, focus is on one shot activity of a single firm/household that doesn't interact with others. The present paper, in contrast, takes into account market-wide

interaction of decision makings as in Fujita (2007) which pays attention to the interaction of

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exporting firms, and Fujita (2008) which takes notice of competition in a stochastic market.

Structure of this paper is as follows. Section 2 lays out a stochastic dynamic Harris-Todaro model and Section 3 derives the market equilibrium. Based on these analyses, Section 4 reveals the relationship between the environmental fluctuation and migration in the stochastic dynamic Harris-Todaro economy. Concluding remarks are made in Section 5.

It is revealed that if uncertainty of environment in the rural area increases, inhabitants in the rural area should stay there longer, while, if the vulnerability of environment in the rural area increases, inhabitants in the rural area should leave there earlier, where uncertainty and vulnerability are respectively defined as the variance of stochastic fluctuation of environmental stock and the degree of sudden reduction of the environmental stock caused by some shock that happens from time to time.

## BASIC MODEL

Let us consider an inter-temporal economy, which is populated by a continuum of inhabitants with a mass of $L$. The economy in the present paper consists of rural and urban areas, where every inhabitant is in the rural area at first and considers every time whether to stay there or move to the urban area. We assume that labour supply of each inhabitant is unity and, as in the original Harris-Todaro model, we exclude the possibility that an inhabitant supplies simultaneously her/his labour to both areas. Time passes continuously and the time horizon is infinite, with importance of the future diminishing as time passes, which we capture by discount rate $\rho$.

We assume that environmental stock of rural sector, $S$, follows a stochastic process of (1).

$$\frac{dS}{S}=\sigma dz\text{-}Sdq, \qquad (1)$$

where $\sigma$, $dz$ and $dq$ are variance, Winner process and Poisson process of the environmental stock of the rural sector, respectively. Following Chakraborty (2015) that points out the vulnerability of the environment, we assume that the environment in this model is vulnerable, in that $S$ falls by a fixed percentage $\eta$ (with $0<\eta<1$) with a small probability $\lambda dt$. Throughout this paper, we define uncertainty and vulnerability as increases in $\sigma$ and $\eta$, respectively. As for the environmental stock in the urban sector, on the other hand, we specify it to be unity, in order to simplify the analysis.

We assume inhabitants in the rural area are owner farmers, who sell their agricultural products at the price of unity and buy the manufactured products at the price of $p$, which are produced by one firm in the urban area, hiring urban workers at the wage of $w$ who are immigrants from the rural area. We assume that $p$ is the world price that is constant over time. As for the technology, we assume that $L_M$ units of labor are transformed into $\varepsilon L_M{}^m$ units of manufactured products, where $\varepsilon$ is a positive constant and $m$ is a constant that satisfies $0<m<1$. Then, profit of the firm, $\pi$, is expressed as $\pi=p\varepsilon L_M{}^m\text{-}wL_M$, and hence, labor demand for production of the manufactured products is determined as

$$L_M=\left(\frac{p\varepsilon m}{w}\right)^{\frac{1}{1-m}}, \qquad (2)$$

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from the first order condition. As (2) shows, not every inhabitant is employed in the urban area. Thus, if we let $L_A$ denote the number of owner farmers who stay in the rural area, probability of being hired in the urban area is $\frac{L_M}{L-L_A}$, and hence, expected wage of the urban sector is determined as $\frac{wL_M}{L-L_A}$, which becomes $\frac{(p\varepsilon m)^{\frac{1}{1-m}}}{L-L_A}(\frac{1}{w})^{\frac{m}{1-m}}$ by substituting (2). It is assumed that the excess supply of the manufactured products is exported at $p$ for the simplicity of analysis, as in Fukuyama and Naito (2007).

We assume that every inhabitant receives the same utility if she/he stays in the rural area. That is, if she/he consumes $D_A$ and $D_M$ units of agricultural products and manufacturing products in an environment of stock $S$, her/his utility is

$$U_R = (D_A)^{1-h}(D_M)^h S, \tag{3}$$

where $h$ is a parameter that satisfies $0<h<1$.

Utilities in the urban area, on the other hand, are assumed to differ from each other. That is, if $kth$ inhabitant consumes $D_A$ and $D_M$ units of agricultural products and manufacturing products, her/his utility is

$$U_U(k) = (D_A)^{1-h}(D_M)^h k. \tag{4}$$

We assume that $k$ distributes uniformly on $]0,L]$ with one density for each.

Since budget constraints for each inhabitant are

$$D_A + pD_M = 1 \quad \text{if in the rural sector;} \tag{5}$$

$$D_A + pD_M = \frac{wL_M}{L-L_A} \quad \text{if in the urban sector,} \tag{6}$$

from the maximization of utilities subject to the above two budget constraints, we obtain the following inverse demand function for the manufacturing products for each inhabitant no matter where she/he lives:

$$p(D_A, D_M) = \frac{h}{1-h}\frac{D_A}{D_M}. \tag{7}$$

Using (5) - (7) to eliminate $D_A$ and $D_M$ in (3) and (4), we have the utility of the $kth$ inhabitant as

$$U_R = \left(\frac{h}{p}\right)^h (1-h)^{1-h} S \quad \text{if in the rural sector;} \tag{8}$$

$$U_U(k) = \left(\frac{h}{p}\right)^h (1-h)^{1-h} \frac{wL_M}{L-L_A} k \quad \text{if in the urban sector.} \tag{9}$$

The present paper assumes that each inhabitant determines her/his timing of migration anticipating the urban utility. Since the model of the present paper is stochastic, the optimal timing is expressed by cut off level of the urban utility. That is, the problem for the $kth$ inhabitant to solve is to determine her/his cut off urban utility level, $U(k)^*$, in a sense she/he moves to the urban sector if $U_U \geqq U(k)^*$.

Conjecture of each inhabitant is assumed to interact in the following way. If the $k$th inhabitant moves to the urban area, those whose utility is more than that in the urban area, *i.e.*, inhabitant $j\in[k,L]$, move to the urban area, which means total inhabitant in the urban area is $L$-$k$, and hence total inhabitant in the rural area is $k$, *i.e.*, $k=L_A$. In what follows, we define the $kth$ inhabitant to be marginal if inhabitants $j\in[k,L]$ are

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in the urban area. It is also assumed that inhabitant's decision making is complete when she/he moves to the urban sector, and after working in the urban sector she/he returns to the rural sector and formulates/solves the optimal stopping problem again from the beginning.

## OPTIMAL TIMING OF THE MIGRATION

Based on the above analysis, in this section we derive the optimal timing of the migration.

First of all, by making use of Ito's lemma, we can express the stochastic process of the utilities in the rural area as

$$\frac{dU_R}{U_R} = \sigma dz - U_R dq, \qquad (10)$$

with the initial value of $U_R(0) = \left(\frac{h}{p}\right)^h (1-h)^{1-h} S(0)$, where $S(0)$ is the initial value of environmental stock in the rural area.

Now, what the $k$th inhabitant maximizes is the discount sum of expected utilities after moving to the rural area net of opportunity cost (*i.e.*, utility in the rural sector). Thus, the problem of the *kth* inhabitant should solve is formulated as:

$$Max \quad e^{-\rho t} \left\{ \left(\frac{h}{p}\right)^h (1-h)^{1-h} \frac{(p\varepsilon m)^{\frac{1}{1-m}}}{L-L_A} \left(\frac{1}{w}\right)^{\frac{m}{1-m}} k - U_R \right\} \qquad (11)$$

*Subject to*

$$\frac{dU_R}{U_R} = \sigma dz - U_R dq;$$

$$U_R(0) = \left(\frac{h}{p}\right)^h (1-h)^{1-h} S(0).$$

Through the standard procedure of dynamic programming (e.g. Dixit and Pyndick (1994)), $U(k)^*$ is determined as:

$$U(k)^* = \frac{\alpha}{\alpha-1} \left(\frac{h}{p}\right)^h (1-h)^{1-h} \frac{(p\varepsilon m)^{\frac{1}{1-m}}}{L-L_A} \left(\frac{1}{w}\right)^{\frac{m}{1-m}} k, \qquad (12)$$

where $\alpha$ is the positive solution to the simultaneous equations $F(x) = \frac{1}{2}\sigma^2 x(x-1)$ and $f(x) = (\rho+\lambda) - \lambda(1-\eta)^x$ as in Figure1 *i.e.*, the positive solution to the characteristic equation $\frac{1}{2}\sigma^2 x(x-1) - (\rho+\lambda) + \lambda(1-\eta)^x = 0$. (See the Appendix for more detail.) Note that it is clear that $\alpha > 1$.

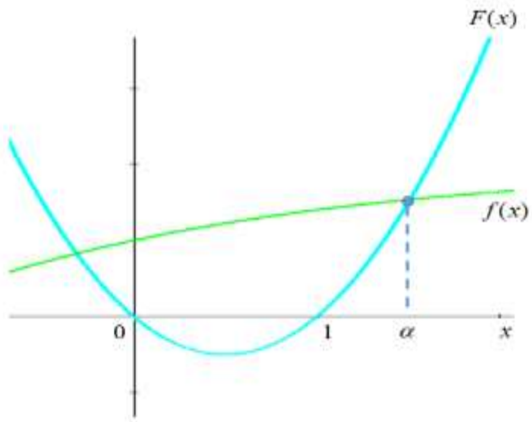


Figure 1 determination of $\alpha$.

The above discussion means that the $k$th inhabitant should move to the urban area if $U_R$ becomes the level of (12). This situation is expressed by the following equation (13), which is obtained by equating (12) with (8) and substituting $L_A = k$;

$$\left(\frac{h}{p}\right)^h (1-h)^{1-h} S = \frac{\alpha}{\alpha-1} \left(\frac{h}{p}\right)^h (1-h)^{1-h} p\varepsilon m^{\frac{1}{1-m}} \left(\frac{1}{w}\right)^{\frac{m}{1-m}} \frac{k}{L-k}. \qquad (13)$$

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Thus, by solving (13) with respect to $k$, the equilibrium marginal inhabitant $k^*$ (which is the equilibrium number of inhabitant in the rural area, $L_A^*$) is determined as

$$k^* = \frac{L}{\frac{\alpha}{\alpha-1}\frac{1}{S}p\varepsilon m^{\frac{1}{1-m}}(\frac{1}{w})^{\frac{m}{1-m}}+1}. \qquad (14)$$

With this, the model closes, which enables us to examine the characteristics of the migration in the Harris-Todaro economy with stochastic fluctuation of environmental stock.

First of all, as Figure 2 shows, increase in $\sigma$ shifts $F(x)$ inward to $\tilde{F}(x)$, to decrease $\alpha$ to $\alpha'$, which brings about increase in $\frac{\alpha}{\alpha-1}$, and hence, decrease in $k^*$. Since $k^*$ is the number of inhabitants who stay in the rural area, we have the following proposition.

***Proposition1:*** *If $\sigma$ decreases, number of inhabitants who stay in the rural area increases.*

This proposition implies that if uncertainty of environment increases, inhabitants in the rural area should stay there longer, since the value of waiting increases.

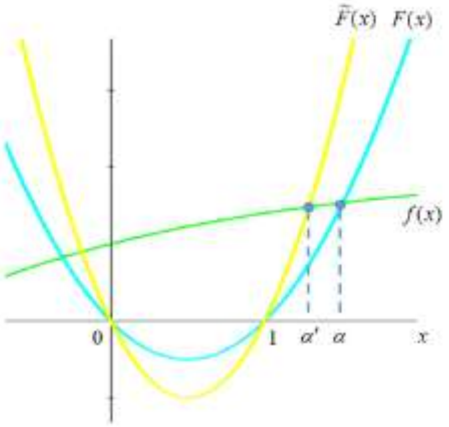


Figure 2 effect of an increase in $\sigma$ on $\alpha$.

Similarly, as Figure 3 shows, increase in $\eta$ shifts $f(x)$ upward to $\tilde{f}(x)$, to increase $\alpha$ to $\alpha''$, which brings about decrease in $\frac{\alpha}{\alpha-1}$, and hence, increase in $k^*$. Thus, as proposition 1, we have the following proposition.

***Proposition2:*** *If $\eta$ increases, number of inhabitants who stay in the rural area decreases.*

This proposition says if the vulnerability of environment increases, contrary to proposition 1, inhabitants in the rural area should leave there earlier.

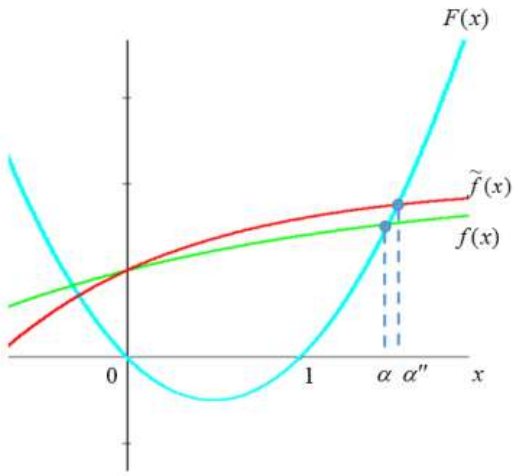


Figure 3 effect of an increase in $\eta$ on $\alpha$.

## CONCLUDING REMARKS

In this paper, we extended Harris-Todaro model and examined the properties of migration in a stochastically fluctuating economy. Main results of this paper are: (1) if uncertainty of environment in the rural area increases, inhabitants in the rural area should stay there longer, while (2) if the vulnerability of environment in the rural area increases, contrary to (1), inhabitants in the rural area should leave there earlier.

It is necessary to examine the robustness of the results by assuming more general utility function and production function. It is also necessary to relaxing the assumption of uniform distribution of inhabitants. We take up such analysis next.

## REFERENCES

[1] Baldursson, F. and I. Karatzas.1997. Irreversible Investment and Industry Equilibrium, *Finance and Stochastics* vol1. 69-90.

[2] Banergee, B. 1983. The Role of the Informal Sector in the Migration Process: a Test of Probabilistic Migration Models and Labour Market Segmentation for India, *Oxford Economic Papers*, New Series, vol.35(3), 399-422.

[3] Bentolila, S. and G. Bertola. 1990 Firing Costs and Labor Demand: How Bad is Eurosclerosis, *Review of Economic Studies*, 57(3), 381-402.

[4] Brueckner, J.K. 1990. Analyzing Third World Urbanization: A Model with Empirical Evidence, *Economic Development and Cultural Change*, 38, 587-610.

[5] Brueckner, J.K. and H-A. Kim. 2001. Land Markets in the Harris-Todaro Model: A New Factor Equilibrating Rural-Urban Migration, *Journal of Regional Science*, 41, 507-520.

[6] Brueckner, J.K. and Y. Zenou. 1999. Harris-Todaro Models with a Land Market, *Regional Science and Urban Economics*, 29, 317-339.

[7] Caballero, R. and R. Pindyck. 1996. Uncertainty, Investment, and Industry Evolution, *International Economic Review*, Vol.37(3), 641-662.

[8] Chakraborty, R. 2015. Importance of Using Renewable Sources of Energy for Maintaining Ecological Sustainability In Case of in Extreme Climate Vulnerable Zones : A Case Study of Sagar Island of Sundarban, *International Journal of Management and Economics Invention*, Vol1(6), 247-253.

[9] Dixit, A.K. 1989 Hysteresis, Import Penetration, and Exchange Rate Pass Through, *Quarterly Journal of Economics* 104, pp.205-228.

[10] Dixit, A. and R. Pindyck. 1994 *Investment under Uncertainty*. Princeton University Press, Princeton, New Jersey,

[11] Farzin, Y. H., K.J.M.Huisman and P.M.Kort. 1988 Optimal Timing of Technology Adoption, *Journal of Economic Dynamics and Control* 22, pp.779-799

[12] Fujita, Y. 2007 Toward a New Modeling of International Economics: An Attempt to Reformulate an International Trade Model Based on Real Option Theory. *Physica A: Statistical Mechanics and its Applications*, 383(2), pp.507-512

[13] Fujita, Y. 2008 Competition and Welfare for a Stochastically Fluctuating Market with Irreversible Decisions. *Physica A: Statistical Mechanics and its Applications*, 387(12), 2846-2850.

[14] Fukuyama, H. and T. Naito, 2007, "Unemployment, Trans-boundary Pollution

||Volume||1||Issue||07||Pages-331-373||Aug-2015|| ISSN (e): 2395-7220
www.rajournals.in DOI : 10.18535/ijmei/v1i7.01

and Environmental Policy in a Dualistic Economy," *Review of Urban and Regional Development Studies*, Vol.19(2), 154-172.

[15] Harris J. and M. P.Todaro. 1970. Migration and Unemployment: a Two-Sector Analysis, *American Economic Review* Vol.60, 126-142.

[16] Jones D. and R.O'Neill. 1995. Development Policies, Urban Unemployment and Deforestation: the Role of Infrastructure and Tax in a Two-Sector model, *Journal of Regional Science*, Vol.35(1), 135-153.

[17] Leahy, J.1993. Investment in Competitive Equilibrium: the Optimality of Myopic Behavior, *Quarterly Journal of Economics*, Vol.108(4), 1105-1133.

[18] McDonald, R. and Siegel, D. 1986 The Value of Waiting to Invest, *The Quarterly Journal of Economics*, 101, 707- 727.

[19] Nagakome, M. 1989. Urban Unemployment and the Spatial Structure of Labor Markets: an Examination of the Todaro Paradox in a Spatial Context, *Journal of Regional Science*, 29, 161-170.

[20] Todaro M. 1969. A model of Labour Migration and Urban Unemployment in Less Development Countries, *American Economic Review* Vol.59, 138-148.

## APPENDIX

First of all, let $V(k)$ denote the vale which is derived by substituting the solution to (11) into the object function, $e^{-\rho t}$ { $\left(\frac{h}{p}\right)^h (1-h)^{1-h} \frac{(p\varepsilon m)^{\frac{1}{1-m}}}{L-L_A} (\frac{1}{w})^{\frac{m}{1-m}} k - U_R$}, we obtain the Bellmann equation for each inhabitant as $\rho Vdt = E(dV)$, then we can expand $dV$ as

$$dV = \frac{\partial V}{\partial U_R} dU_R + \frac{1}{2}\frac{\partial^2 V}{\partial {U_R}^2}(dU_R)^2,$$

by using Ito's lemma. If we substitute the stochastic process of $U_D$ into this formula, we have

$$E(dV) = \frac{\sigma {U_R}^2}{2}\frac{\partial^2 V}{\partial {U_R}^2} dt,$$

And by substituting Bellmann equation into this formula, we have

$$\frac{\sigma {U_R}^2}{2}\frac{\partial^2 V}{\partial {U_R}^2} - \rho V(U_R) = 0$$

Boundary condition, value matching condition and smooth pasting condition are expressed, respectively, as

$$V(\infty) = 0$$

$$V(U(k)^*) = (\frac{w}{p})^{1-h} k \frac{L_T}{L-L_D} - U(k)^*$$

$$\left.\frac{dV}{dU_R}\right|_{U_R = U(k)^*} = 1$$

Thus, the solution is expressed as

$$F(U_R) = A{U_R}^{\alpha},$$

where $A$ is a constant and $\alpha$ is the positive solution to the characteristic equation $\frac{1}{2}\sigma^2 x(x-1) - (\rho+\lambda)+\lambda(1-\eta)^x = 0$.

Thus, $\alpha$ is determined graphically as in Figure 1.