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Variant Multi Objective Tsp Model

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Abstract: ***Let there be a set of*** $'n'$ ***cities. Pair of distances between the cities are known and distance between*** i^{th} ***cities to*** j^{th} ***city is given. The cost of travel between pair of cities is given. The cost of*** $C(i, j)$ ***and Distance*** $D(i, j)$ ***need not be related. These two matrices*** $C(i, j)$ ***and*** $D(i, j)$ ***can be simply write as C and D respectively. The travelling salesman starts his business from head quarter city 1 and he wants to travel*** n_0 ***cities less than*** n***.The travelling salesman in his tour visits*** n_0 ***cities with minimum distances then the same route need not be with minimum costs and same way a route with minimum costs need not be with a minimum distance. The cost and distance are different unrelated factors but the salesman wants to minimize in both cases as much as possible. The objective is he wants to visit*** n_0 ***cities nearly with minimum cost and minimum distance. Both the factors distance and cost being independent, absolutely in both cases minimum is not possible. The salesman may be interested in one factor when compared with the other factor or vice versa. We want to suggest tours where both factors are considered and considerable minimum tours are planned, with different constraints under considerations.***
Keywords: travelling salesman, tour, distance, cost, head quarter

INTRODUCTION:

Travelling salesman problem (TSP) is a very popular model. The traveling salesman problem involves of a salesman and a set of cities. The salesman has to visit each one of the cities starting from a hometown and returning to the same city. The challenge of the problem is that the traveling salesman wants to minimize the total length of the trip. So many researchers developed different algorithms. The earliest papers on the Generalized Travelling Salesman Problem discuss the problem in the context of particular applications by Henry-Labordere [1], Saskena [6], and Srivasatava et al [10]. Fischetti et al [9] use the polyhedral results to develop a branch and cut algorithm (A lagrangian-based branch and bound algorithm] FatihTasgetiren et al.[2] and Dimitrijevic [5] discussed the variant travelling salesman problem.

Only a few authors Laporte et al [7] handle fixed costs for including a node on the tour, possibly because such fixed costs can be incorporated into the distance costs via a simple transformation.Laporte[8] describes An overview of Exact and Appropriate algorithm.

Madhu Mohan Reddy et al.[11] describes aproblem called an alternate travelling salesman problem. The objective of this problem is to find a Tour such that total length of the tour minimum. An exact algorithm is proposed for this TSP. The algorithm solves the problem on identify the key patterns which optimize the objective of thecost/distance. Bhavani and Sundara Murthy [3] explained Time-Dependent Traveling Salesman Problem.Ben-Arieh et al. [4] explained Process Planning for rotational parts using the generalized TSP.

Problem Description:

Letset of $'n'$ cities be $N = \{1,2,3,\ldots,n\}$. Pair of distance between the cities is known and let it be $D(i, j)$, i.e. distance between i^{th} cities to j^{th} city is given. The cost of travel between pair of cities $C(i, j)$ is given. The cost of $C(i, j)$ and Distance

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$D(i, j)$ need not be related. These two matrices $C(i, j)$ and $D(i, j)$ can be simply write as C and D respectively. The travelling salesman starts his business in head quarter city 1 and he wants to travel n_0 cities less than n .The travelling salesman in his tour visits n_0 cities with minimum distancesthen the route need not be with minimum costs and same way a route with minimum costs need not be with a minimum distance when it is with minimum cost. The cost and distance are different unrelated factors but the salesman wants to minimize in both cases as much as possible. The objective is he wants to visit n_0 cities mostly with minimum cost and minimum distance. Both the factors distance and cost being independent, absolutely in both cases minimum is not possible. The salesman may be interested in one factor when compared with the other factor or vice versa. We want to suggest tours where both factors with considerable minimum tour are planned.

The travelling salesman wants a tour of n_0 cities with the following plans.

1 A tour where the average of distance and cost be minimum. The values of the tour distance and costs be (DV_1, CV_1) and the set of cities is N_1, $|N_1| = n_0$.

2 A tour with minimum distance irrespective of cost. The values of the tour distance and costs be (DV_2, CV_2) and the set of cities is N_2, $|N_2| = n_0$.

3 A tour with minimum cost irrespective of distance. The values of the tour distance and costs be (DV_3, CV_3) and the set of cities is N_3, $|N_3| = n_0$.

4 A tour where the distance is $\beta = 20\%$ (say) more than the minimum distance and the cost is correspondingly minimum. The values of the tour distance and costs be (DV_4, CV_4) and the set of cities is N_4, $|N_4| = n_0$.

5 A tour where the cost is $\beta = 20\%$ more than the minimum cost and the distance is correspondingly minimum. The values of the tour distance and costs be (DV_5, CV_5) and the set of cities is N_5, $|N_5| = n_0$.

The set of 5 values involving in total distance and total costs for the above 5 tours is given to the customer for his choice.

So, the objective of the problem is to identify the above 5 sets of tour values which involves the tour of n_0 cities.

3 MATHEMATICAL FORMULATION:

Let the tour of n_0 cities be $1 \rightarrow \alpha_1 \rightarrow \alpha_2 - - - \alpha_{n_0 - 1} \rightarrow \mathbf{1}$, $\alpha_i \in N$

Let X be a (0, 1) indicator matrix such that $x(\alpha_i, \alpha_{i+1}) = 1 and 0$, for other pair of values.

This X also represents the above tour of n_0 cities.

$\alpha_i \in N_0,\ i = 1,2,\ldots n_0 - 1\ ,1 \in N_0$

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Let $E(i,j)=\frac{1}{2}[D(i,j)+C(i,j)]\ldots(1)$

Let X represents tour of n_0 cities.

Then $Z=\sum_i\sum_j E(i,j)\,X(i,j),\ i,j\in N_0 and\ N_0\subset N$ …(2)

The objective is to find X such that

$$\underset{X}{Min}\,Z(X)=Z_1(X_1) \quad ...(3)$$

$$Z_{1D}=\sum_i\sum_j D(i,j)\,X_1(i,j) \quad ...(4)$$

$$Z_{1C}=\sum_i\sum_j C(i,j)\,X_1(i,j)\,. \quad ...(5)$$

Consider the matrix D alone.

$$Z(X)=\sum_i\sum_j D(i,j)\,X(i,j) \quad ...(6)$$

$$\underset{(X)}{Min}\,Z(X)=Z_2(X_2)=Z_{2D} \quad ...(6.1)$$

$$Z_{2C}=\sum_i\sum_j C(i,j)\,X_2(i,j) \quad ...(7)$$

Similarly consider the matrix C alone.

$$Z(X)=\sum_i\sum_j C(i,j)\,X(i,j) \quad ...(8)$$

$$\underset{(X)}{Min}\,Z(X)=Z_3(X_3)=Z_{3c} \quad ...(9)$$

$$Z_{3D}=\sum_i\sum_j D(i,j)\,x_3(i,j) \quad ...(10)$$

$$Z_4(X_4)=\underset{X}{Min}\sum_i\sum_j C(i,j)\,X(i,j)=Z_{4c} \quad ...(11)$$

$$Z_5(X_5)=\underset{X}{Min}\sum_i\sum_j D(i,j)\,X(i,j)=Z_{5D} \quad ...(12)$$

In Equation (1), E (i, j) represents that average values of corresponding distance matrix D (i, j) and cost matrix C (i, j) respectively.

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Equation (2) represents that the problem to find total value in a tour with respect to the matrix E(i ,j).

In equation (3), $Z(X)$ is minimum and let it be X_1, where X_1 gives the tour with minimum average corresponding values of both distance and cost matrices.

In Equation (4), Z_{1D} is the total distance of the tour represented by tour X_1 with respect to the matrix D.

In Equation (5), Z_{1C} is the total cost of the tour represented by tour X_1 with respect to the matrix C.

Equation (6) represents that the problem to find total distance value in a tour with respect to the matrix D.

In equation (6.1), $Z(X)$ is minimum and let it be X_2, where X_2 gives the tour with minimum distance with respect to the matrix D.

In Equation (7) Z_{2c} is the total cost in a tour represented by X_2 with respect to the matrix C.

Equation (8) represents that the problem to find total cost value in a tour with respect to the matrix C.

Equation (9) represents $Z(X)$ is minimum and let it be X_3, where X_3 gives the tour with minimum cost with respect to the matrix C.

In Equation (10), Z_{3c} is the total cost in a tour represented by X_3 with respect to the matrix D.

Equation (11) represents X_4 is the tour with $Z_{4D} \le Z_2(D) + \beta Z_2(D)$

Equation(12) represents X_5 is the tour with $Z_{5C} \le Z_3(C) + \beta Z_3(C)$

4 NUMERICAL EXAMPLE:

The algorithm and concepts are developed and illustrated by a numerical example. For which the total number of cities N = {1, 2, 3, 4, 5, 6, 7}. For the following example, first we find the optimal solution in Lexi-Search approach using the "Pattern Recognition Technique". Here city "1"is taken as the head quarter. In the table the distance (i, i) is taken as "-"indicates dis-connectivity to the corresponding cities, which can also be taken as ∞. Here the entries $D(i, j)$, $C(i, j)$ and $E(i, j)$ taken as non-negative integers, it can be easily seen that this is not a necessary condition and the distance, cost be any positive quantity.

Table-1

Distance matrix: $[D(i, j)]$

-	8	13	1	8	11	1
5	-	26	1	3	11	10
7	9	-	18	4	6	16
8	1	14	-	2	12	17
14	6	14	12	-	5	3
15	18	1	14	19	-	12
2	12	6	19	2	16	-

Table-2

Cost matrix: $[C(i, j)]$

-	8	15	1	10	15	1
7	-	14	3	11	13	2
3	5	-	20	4	10	14
10	1	12	-	2	10	15
6	8	16	20	-	3	5
7	16	3	20	17	-	22
10	8	4	17	4	20	-

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Table-3Average of distance and cost matrix: [$E(i, j)$]

-	8	14	1	9	13	1
6	-	20	2	7	12	6
5	7	-	19	4	8	15
9	2	14	-	2	11	16
10	7	15	16	-	4	4
11	17	2	17	18	-	17
6	10	5	18	3	18	-

From the above table-1 $D(2,5)$, means the distance of the connecting the city 2 to 5 is 3. From the above table-2, $C(3,7)$ =14, means the cost of the connecting the city 3 to 7 is 14. From the above table-3 $E(6,3)$ =2, means the average of $D(6,3)+C(6,3)$. The objective is of the problem is to identify the above 5 sets of values which involves the tour of $n_0 = 5$ cities, salesman wants to visit with considerable minimum distance and cost is given below.

1. A tour where the average of distance and cost be minimum. The values of the tour distance and costs be (DV_1, CV_1) and the set is in N_1, $|N_1| = 5$.
2. A tour with minimum distance irrespective of cost. The values of the tour distance and costs be (DV_2, CV_2) and the set is in N_2, $|N_2| = 5$.
3. A tour with minimum cost irrespective of distance. The values of the tour distance and costs be (DV_3, CV_3) and the set is in N_3, $|N_3| = 5$.
4. A tour where the distance is $\beta = 20\%$ (say) more than the minimum distance and the cost is correspondingly minimum. The values of the tour distance and costs be (DV_4, CV_4) and the set is in N_4, $|N_4| = 5$.
5. A tour where the cost is $\beta = 20\%$ more than the minimum cost and the distance is correspondingly minimum. The values of the tour distance and costs be (DV_5, CV_5) and the set is in N_5, $|N_5| = 5$.

5 CONCEPTS AND DEFINITIONS

5.1 Definition of a pattern

An indicator two-dimensional array which is associated with a tour is called a 'pattern'. A pattern is said to be feasible if X is a feasible tour.

V (X) = ΣΣ D (i, j) X (i, j)

5.2 Feasible solution of the problem

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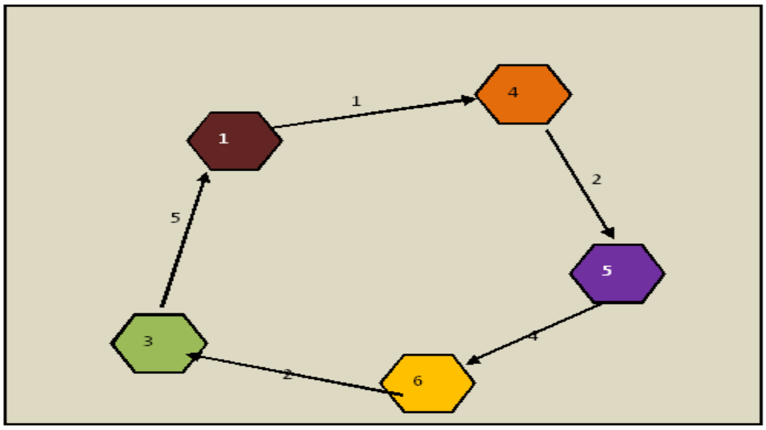
Consider an ordered pair set{(1,4),(4,5),(5,6),(6,3),(3,1)}and it represents the tour $1 \to 4 \to 5 \to 6 \to 3 \to 1$, which is a feasible solution for average of distance and cost matrix E.The Figure-1, represents the feasible solution of the above ordered pairs. In the graphs (figure-1, figure-2, figure-3, figure-4)hexagons represents cities and the value in hexagons indicates the name of the respective cities also values at each arc representcorresponding average value of costand distance matrix E.

Figure-1



From the Figure-1, the city 1 connected to 4, city 4 connected to 5, city 5 connected to 6, city 6 connected to 3 and city 3 connected to 1. Here, the feasible solution is 14.Average of total cost and distance =E(1,4)+E(4,5)+E(5,6)+E(6,3)+E(3,1)
=1+2+4+2+5=14.

From the solution of average of distance and cost matrix E(Figure-1), we can split the corresponding solutions from the distance matrix(table-1) and cost matrix(table-2)is given in figure-2.

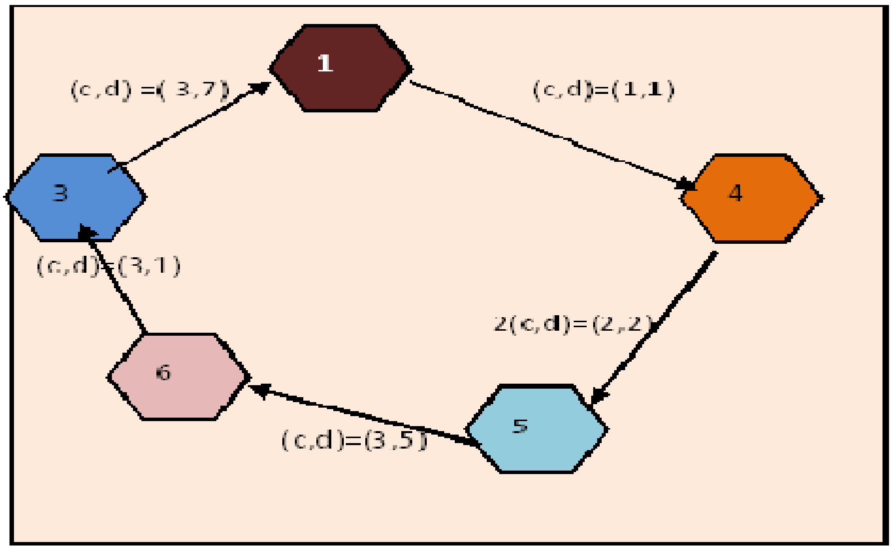


figure-2.

Average of total cost and distance =E(1,4)+E(4,5)+E(5,6)+E(6,3)+E(3,1)=1+2+4+2+5=**14**

Total cost=C(1,4)+C(4,5)+C(5,6)+C(6,3)+C(3,1)=1+2+3+3+3=12.

Total distance=D(1,4)+D(4,5)+D(5,6)+D(6,3)+D(3,1)=1+2+5+1+7=16.

From the Figure-2, the city 1 connected to 4, city 4 connected to 5, city 5 connected to 6, city 6 connected to 3 and city 3 connected to 1. Values on arcs represent corresponding cost and distances betweennodes respectively.

Total distance=D(1,4)+D(4,5)+D(5,6)+D(6,3)+D(3,1)=1+2+5+1+7=16(from table-1). Total cost=C(1,4)+C(4,5)+C(5,6)+C(6,3)+C(3,1)=1+2+3+3+3=12 (from table-2).

Let the total feasible distance be $DV_1 = 16$ and the total feasible cost is $CV_1 = 12$.

Therefore, the values of the tour distance and costs be $(DV_1, CV_1) = (16,12)$ and the number cities $|N_1| = 5$.

6Solution Approach: In the above figure-1,2 for the feasible solution we observe that there are 5 ordered pairs taken along with the value from the average of distance and cost matrix $[E(i,j)]$, distance matrix $[D(i,j)]$, cost matrix $[C(i,j)]$ of the numerical example in table-1,2,3 respectively. The 5 ordered pairs are selected such that they represent a feasible solution in figure-1,2,3. So the problem is that we have to select 5 ordered pairs from the distance matrix along with values such that the total value is minimum and represents a feasible solution. For this selection of 5 ordered pairs we arrange the 7×7=49 ordered pairs in the increasing order and call this formation as alphabet table and we will develop an algorithm for the selection along with the checking for the feasibility.

6.1Alphabet table (partial) for average of distance and cost matrix E(i,j):

There is m= n x n ordered pairs in the two-dimensional array X. Let E be the corresponding array of average of distance and cost matrix. For convenience these are arranged in ascending order of their correspondingmatrix E(i,j) and are indexed from 1 to k(SundaraMurthy-1979). Let SN= [1, 2, 3… k] be the set of k indices. If a, b∈ SN and a < b then E (a) $\leq$E (b). Also let the arrays R and C be the array of row and column indices of the ordered pairs are represented by SN and CE be the array of cumulative sum of the elements of E. The arrays SN, E, CE, R, C, for the numerical example are given in the Table-3. If p∈ SN then (R(p),C(p)) is the ordered pairs and E(a)=E(R(a),C(a)) is the value of the ordered pairs and CE (a) $= \sum_{i=1}^{a} E(i)$.

Table-4

S.N	**E**	**CE**	**R**	**C**
1	1	1	1	4
2	1	2	1	7
3	2	4	2	4
4	2	6	4	2
5	2	8	4	5
6	2	10	6	3
7	**3**	**13**	**7**	**5**
8	4	17	3	5
9	4	21	5	6
10	4	25	5	7

The first column represents serial number, second column represents the increasing order of the values of $E(i,j)$,third column represents the cumulative of $E(i,j)$, fourth and fifth columnsrepresent the corresponding row and column of E.

Let us consider 7€SN. It represents the ordered pair (R (7), C (7)) = (3, 5). Then E (7) =E(3, 5) =4 andCE (3, 5) =17.

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6.2 Definition of a word

Let SN = (1,2,…) be the set of indices, E be an array of corresponding distances of the ordered pairs and Cumulative sums of elements in E is represented as an array CE. Let arrays R, C be respectively the row, column indices of the ordered pairs. Let $L_k = \{a_1, a_2, - - -- - , a_k\}$, $a_i \in$ SN be an ordered sequence of k indices from SN. The pattern is represented by the ordered pairs whose indices are given by L_k is independent of the order of a_i in the sequence. Hence for uniqueness the indices are arranged in the increasing order such that $a_i \leq a_{i+1}$, i = 1, 2, - - - -, k-1.

6.3 Value of the word

The value of the partial word L_k, V (L_k) is defined recursively as V (L_k) = V(L_{k-1}) +E (a_k) with V (L_o) = 0 where E (a_k) is the distance/cost array arranged such that E (a_k) < E (a_{k+1}). V (L_k) and V(x) the values of the pattern X will be the same. X is the (partial) pattern represented by L_k, (Sandara Murthy – 1979).

6.4 Feasibility criterion of a partial word

An algorithm was developed, in order to check the feasibility of a partial word $L_{k+1} = (a_1, a_2, - - - -- a_k, a_{k+1})$ given that L_k is a feasible partial word. We will introduce some more notations which will be useful in the sequel.

6.5Algorithm-1 (Feasible checking)

STEP1 : IX=0

STEP 2 : IS (IR (RA) = 1) IF YES GOTO 8

IF NO GOTO 3

STEP3 : W = CA GOTO 4

STEP 4 : IS W = RA IF YES GOTO 8

IF NO GOTO 5

STEP5 : IS SW (W) = 0 IF YES GOTO 7

IF NO GOTO 6

STEP 6 : W = SW (W) GOTO 4

STEP7 : IX=1 IF YES GOTO 8
IF NO GOTO 1

STEP 8 : STOP

We start with the partial word $L_1 = (a_1) = (1)$. A partial word L_p is constructed as $L_p = L_{p-1} * (\alpha_p)$. Where * indicates chain formulation. We will calculate the values of V (L_p) and LB (L_p) simultaneously. Then two situations arises one for branching and other for continuing the search.

6.6. Algorithm- 2: (Lexi - Search algorithm)

STEP 1 : (Initialization)

The arrays SN, D, DC, R, C and the values of n, High, MAX are made available IR,SW L, V, LB are initialized to zero. The values I=1, J=0, IR (RA) =0, VT = High

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STEP 2 : J=J+1

IS (J>MAX) IF YES GOTO 11

IF NO GOTO 3

STEP 3 : L (I) = J

RA=R (J)

CA=C (J) GOTO 4

STEP 4 : V (I) =V (I-1) +D (J)

LB (I) =V (I) +CE (J+N-I)-CE (J) GOTO 5

STEP 5 : IS (LB (I)$\geq$VT) IF YES GOTO 10

IF NO GOTO 6

STEP 6 : CHECK THE FEASIBILITY OF L (USING ALGORITHM-1)

IS (IX=0) IF YES GOTO 2

6.7 Search table for average of distance and cost matrix (partial):

The working details of getting an optimal word using the above algorithm for the illustrative numerical example is given in the **Table-5.** The columns named (1), (2), (3),…, gives the letters in the first, second, third and so on places respectively. The columns R, C give the row, column indices of the letter. The last column gives the remarks regarding the acceptability of the partial words. In the following table 8 indicates ACCEPT and R indicates REJECT.

Table-5

S.N	1	2		4	5	V	LB	T	R	C	Remarks
1	1					1	8	2	1	7	A
2		2				2	8	2	1	4	R
3		3				3	10	1	6	3	A
4			4			5	10	3	2	4	R(TR)
5			5			5	12	3	4	5	R(TR)
6			6			6	14	8	7	5	A
7				7		10	14	5	3	5*	R
8				8		10	14	4	5	6	A
9					9	10	14	5	5*	7	R
10					10	15	15	1	3	1	A, VT=15
11				9		10	15*	5	5	7	R (=VT)
12			7			7	15*	5	3	5	R(=VT)
13		4				3	12	3	2	4	A
14			5			5	12	3	4	5	R
15				6		8	12	8	7	5	R
16				7		9	13	5	3	5	R(TR)
17				8		9	13	4	5	6	R
18				9		9	14	5	5	7*	R,(=VT)

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19				10		10	15*	1	3	1	A
20			6			6	14	8	7	5	A
21				7		10	14	5	3	5*	R
22				8		10	14	4	5	6*	R(TR)
23				9		10	15*	5	5	7	R(=VT)
24			7			7	15*	5	3	5	R(=VT)
25		5				3	14	3	4	5	A
26			6			6	14	8	7	5*	R
27			7			7	15*	5	3	5*	R(=VT)
28		6				4	16*	8	7	5	R(>T)
29	2					1	10	2	1	4	A
30		3				3	10	1	6	3	A
31			4			5		3	2	4	R
32			5			5	12	3	4	5	A
33				6		9	13	8	7	5	R
34				7		9	13	5	3	5*	R
35				8		9	13	4	5	6	A
36					9	1	13	5	5	7	R
37					10	14	14	1	3	1	A, VT=14
38				9		9	14*	5	5	7	R(=VT)
39			6			6	14*	8	7	5	R(=VT)
40		4				3	12	3	2	4*	R

The partial word is L_5 = (29, 30, 32, 35, 37) is a feasible partial word.

At the end of the search the current value of the **VT is 14** and it is the value of optimal feasible word. L_5 = (29, 30, 32, 35, 37) given in the **37throw** of the search table. Therefore, value of the optimal solution of the Lexi search algorithm using pattern recognition technique is 14.

The Figure-**1** represents the feasible optimal solution.

i.e. Average of total cost and distance = E(1,4)+ E(4,5) + E(5,6) + E(6,3) + E(3,1)
=1+2+4+2+5=14

From the solution of average of distance and cost matrix E (Figure-1), we can split the corresponding solutions from the distance matrix (table-1) and cost matrix (table-2) is given in figure-2. i.e. Total distance= D(1,4)+D(4,5)+D(5,6)+D(6,3) +D(3,1)=1+2+5+1+7=16 (from table-1).Total cost= C(1,4)+C(4,5)+C(5,6)+C(6,3)+C(3,1)=1+2+3+3+3=12 (from table-2).

Let the total feasible distance be $DV_1 = 16$ and the total feasible cost is $CV_1 = 12$.

Therefore, the values of the tour distance and costs be $(DV_1, CV_1) = (16,12)$ and the number cities $|N_1| = 5$.

7.1 Alphabet table(partial) for distance matrix D(i,j):

Alphabet table is increasing order of the distance matrix**.** It is explained in section7.

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Alphabet Table-6

S.N	D	CD	R	C	Ct
1	1	1	1	4	1
2	1	2	1	7	1
3	1	3	2	4	3
4	1	4	4	2	1
5	1	5	6	3	3
6	2	7	4	5	2
7	2	9	7	1	34
8	2	11	7	5	4
9	3	14	2	5	19
10	3	17	5	7	5
11	5	22	2	1	7

7.2 Searcch Table:

The working details of getting an optimal word using the above algorithm for the illustrative numerical example is given in the **Table-7.** The columns named (1), (2), (3),…, gives the letters in the first, second, third and so on places respectively. The columns R, C give the row, column indices of the letter. The last column gives the remarks regarding the acceptability of the partial words. In the following table 8 indicates ACCEPT and R indicates REJECT.

Searcch Table-7

t	1	2	3	4	5	V	LB	R	C	Ct	Remarks
1	1					1	5	1	4	1	A
2		2				2	5	1*	7	-	R
3		3				2	6	2	4*	-	R
4		4				2	7	4	2	1	A
5			5			3	7	6*	3	-	R
6			6			4	8	4*	5	-	R
7			7			4	9	7	1	10	A
8				8		6	9	7*	5*	-	R
9				9		7	10	2	5	11	A
10					10	10	10	5	7	5	A,VT=10
11				10		7	12*	5	7	-	R(>VT)
12			8			4	10*	7	5	-	R(=VT)
13		5				2	8	6	3	-	A
14			6			4	8	4	5	-	A
15				7		6	8	7*	1	-	R(T)
16				8		6	9	7	5*	-	R

The partial word is L_5 = (1,4,7,9,10) is a feasible partial word.

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At the end of the search the current value of the **VT is 10** and it is the value of optimal feasible word. L_5 = (1,4,7,9,10) given in the **10th row** of the search table. Therefore, value of the optimal solution of the Lexi search algorithm using pattern recognition technique is 10.

The Figure**-3** represents the feasible optimal solution.

The Figure-3, represents the feasible solution. The hexagons represent cities and the value in hexagons indicates the name of the cities also values at each arc represents distance and cost between the respective two nodes from table-1 and table-2 respectively.

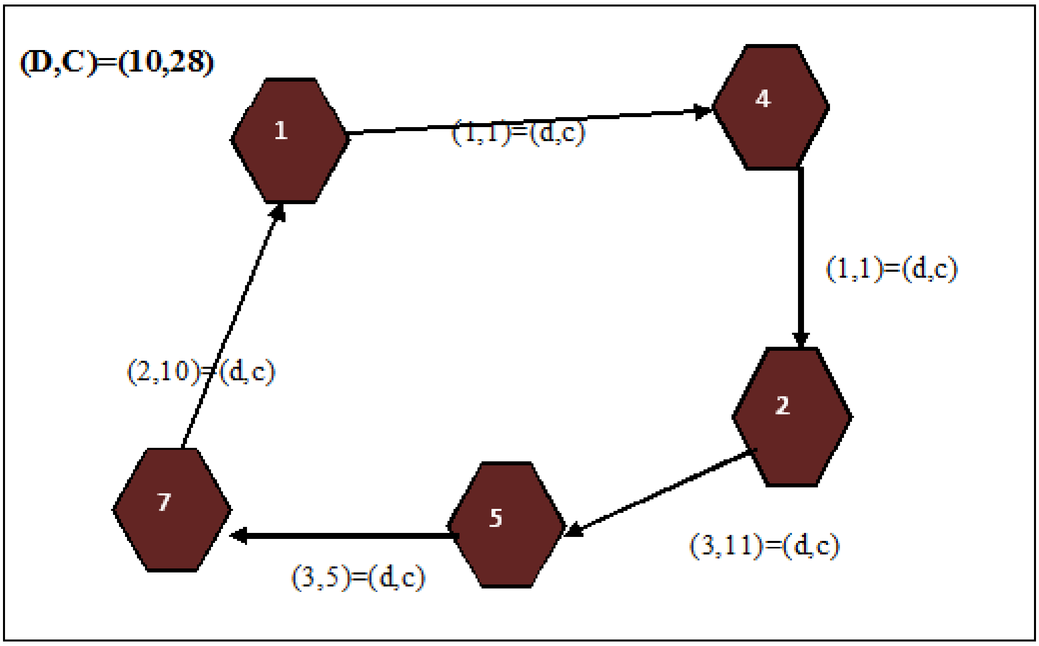

Figure-3

From the above Figure-3, the city 1 connected to 4, city 4 connected to 2, city 2 connected to 5, city 5 connected to 7 and city 7 connected to 1. Here, the feasible solution with respect to cost matrix is 10 and with respect to distance matrix is 28.

Total distance =D(1,4)+D(4,5)+D(5,6)+D(6,3)+D(3,1)=1+1+3+3+2=10and corresponding cost= C(1,4)+C(4,2)+C(2,5)+C(5,7)+C(7,1)=1+1+11+5+10=28.

Let the total feasible distance be $DV_2 = 10$ and the total feasible cost is $CV_2 = 28$.

Therefore, the values of the tour distance and costs be $(DV_2, CV_2) = (10,28)$ and the number cities $|N_1| = 5$.

Let β=20% increase in a optimal distance solution we get 10+2=12. By using the algorithm we get total cost=C(1,4)+C(4,2)+C(2,7)+C(7,3)+C(3,1)=1+1+2+4+3=11 and corresponding distance= D(1,4)+D(4,2)+D(2,7)+D(7,3)+D(3,1)=1+1+10+6+7=25.

Therefore, the values of the tour distances be $(DV_4, CV_4) = (11,25)$ and the number cities $|N_1| = 5$

8 ALPHABET TABLE FOR COST ARRAY(PARTIAL):

Alphabet table is increasing order of the distance matrix**.**

Table-8

S.N	C	CC	R	C	Di
1	1	1	1	4	1
2	1	2	1	7	1
3	1	3	4	2	1
4	2	5	2	7	26
5	2	7	4	5	12
6	3	10	2	4	1
7	3	13	3	1	7
8	3	16	5	6	5
9	3	19	6	3	1
10	4	23	7	3	6
11	5	28	3	2	9
12	5	33	5	7	3
13	6	39	5	1	14

Search table:

The working details of getting an optimal word using the above algorithm for the illustrative numerical example is given in table-9**.** The columns named (1), (2), (3),…, gives the letters in the first, second, third and so on places respectively. The columns R, C give the row, column indices of the letter. The last column gives the remarks regarding the acceptability of the partial words. In the following table 8 indicates ACCEPT and R indicates REJECT.

Table-9

t	1	2	3	4	5	V	LB	R	C	Ct	Remarks
1	1					1	7	1	4	1	A
2		2				2	7	1*	7	-	R
3		3				2	9	4	2	1	A
4			4			4	9	2	7	10	A
5				5		6	9	4*	5	-	R
6				6		7	10	2*	4	-	R
7				7		7	10	3	1	7	A
8					8	10	10	5*	6	-	R(T)
9					9	10	10	6*	3	-	R(T)
8					10	11	11	7	3	6	A(VT=11)
9				8		7	10	5*	6	-	R(T)
10				9		7	11*	6	3	-	R(=VT)
11			5			4	10	4*	5	-	R
12			6			5	11*	2	4	-	R(=VT)
13		4				3	11*	2	7	-	R(=VT)
14	2					1	7	1	7	-	A

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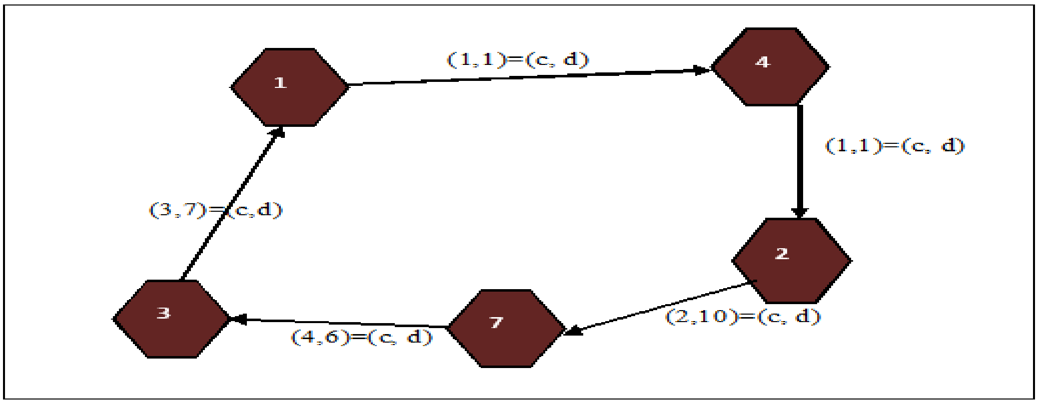
15		**3**				**2**	**7**	**4**	**2**	**-**	**A**
16			**4**			**4**	**9**	**2**	**7**	**-**	**R**
17			**5**			**4**	**10**	**4***	**5**	**-**	**R**
18			**6**			**5**	**11***	**2**	**4**	**-**	**R(VT)**
19		**4**				**3**	**11***	**2**	**7**	**-**	**R(=VT)**
20	**3**					**1**	**11***	**4**	**2**	**-**	**R(VT)**

From the above search table, the optiml solution with respect to cost matrix is 11 and corresponding solution with respect to distance matrix is 25. The Figure-4 represents the above feasible optimal solution. The hexagons represent cities and the value in circles indicates the name of the cities also values at each arc represents cost and distance between the respective two nodes.

Figure-4



From the above Figure-4, the city 1 connected to 4, city 4 connected to 2, city 2 connected to 7, city 7 connected to 3 and city 3 connected to 1. Here, the feasible solution with respect to cost matrix is 11 and corresponding solution with respect to distance matrix is 25. Total cost =C(1,4)+C(4,2)+C(2,7)+C(7,3)+C(3,1)=1+1+2+4+3=11and

Corresponding distance = D(1,4)+D(4,2)+D(2,7)+D(7,3)+D(3,1)=1+1+10+6+7=25.

Let the total feasible distance be $DV_3 = 11$ and the total feasible cost is $CV_3 = 25$.

Therefore, the values of the tour distance and costs be $(DV_3, CV_3) = (10,25)$ and the number of cities $|N_1| = 5$.

9 Conclusion:

In this paper, we have studied a model namely “A Variant Multi Objective TSP model”.. Many researchers studied this travelling salesman problem. We also tried variant travelling salesman problem. In this paper, we take random values of distances and costs which are in table-1 and table-2. The costs and distances need not be related. The corresponding average values of table-1 and table-2 are given in table-3.To find the optimal solution we developed an algorithm. By using this algorithm we find optimal solutions for table-1, table-2 and table-3 are $(DV_2, CV_2) = (10,28)$, $(DV_1, CV_1) = (16,12)$ and $(DV_3, CV_3) = (10,25)$ respectively. If β% increase in optimal solution of distance matrix and cost matrix we get another two pair of values. We will give choice to customer to select any one pair out of 5 solutions.

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