

Optimization of Investment Portfolio Structure Using the Markowitz Model

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ABSTRACT

This study examines the application of the Markowitz model, an innovative portfolio optimization technique developed in the 1950s, in the context of the Russian stock market. The main objectives of this study are to study the theoretical foundations of the Markowitz model, analyze the current state and prospects of the Russian stock market and, ultimately, use the model to build an optimal portfolio structure. By integrating the Markowitz model with the dynamics of the Russian stock market, this study aims to contribute to the understanding of portfolio management and offer practical recommendations to portfolio managers and investors. The conclusions and recommendations obtained from the study will provide valuable information about the effectiveness and suitability of the Markowitz model in the Russian financial landscape. The study used Excel and Google Colab for comprehensive data analysis and portfolio optimization, providing a robust and data-driven approach.

KEYWORDS: Expected return, Investment risks, Portfolio optimization, Markowitz model, Russian stock market.

1. INTRODUCTION

Investment portfolio management is a critical task for investors seeking to maximize returns while reducing risk in financial markets. The Markowitz model, developed by Harry Markowitz in the 1950s, revolutionized the field of portfolio optimization by formalizing the trade-off between expected return and risk. This fundamental model continues to be widely used by investment professionals and academics alike. The article discusses the application of the Markowitz portfolio optimization model in the context of the Russian stock market. The Markowitz model is a fundamental approach to investment management that aims to maximize expected return for a given level of risk or minimize risk to achieve a target return.

Through diversification and covariance analysis of asset returns, the Markowitz model allows investors to determine the efficient frontier - a set of optimal portfolios that provide the maximum expected return for a given level of risk. This has significant implications for investment decisions related to asset allocation, risk management and portfolio rebalancing. The study is aimed at achieving three key goals: (1) consideration of the theoretical foundations of portfolio formation and the Markowitz model; (2) analysis of the current state and prospects of the Russian stock market; (3) application of the Markowitz model for constructing and optimizing investment portfolios for the Russian stock

market. The main research methods used in this paper are collecting data, analyzing it using Excel and Google Colab, applying the Markowitz Portfolio Optimization Model and evaluating the performance of the model using various statistical indicators.

2. THEORETICAL ASPECTS OF INVESTMENT PORTFOLIO OPTIMIZATION

A portfolio is a collection of securities, commodities, real estate, cash equivalents or other assets held by an individual or institutional investor. The purpose of a portfolio is to reduce risk through diversification [8].

Thus, a portfolio is simply a collection of one or more different investment assets. Investment assets are assets that investors own primarily for the purpose of investing. In accordance with the Civil Code of the Russian Federation, the following types of securities are allowed for circulation on the Russian stock market [10]: government bonds; bonds; bill of exchange; check, certificates of deposit, savings certificates, bearer bank passbook, bill of lading, shares, privatization securities (voucher), etc.

* **Markowitz model.** Harry Markowitz (1927 - 2023), Nobel Prize-winning economist, is considered the father of modern portfolio theory. The Markowitz method, also known as modern portfolio theory (MPT), is a widely used approach to optimizing asset allocation in an investment

portfolio [10]. Its goal is to maximize expected return for a given level of risk or minimize risk for a given level of expected return. The key principles are: (1) recognition of the trade-off between risk and return; (2) portfolio diversification to reduce overall risk; and (3) determining the efficient frontier of the optimal portfolio. The method uses estimates of expected return, variance and covariance to determine the optimal asset allocation [5].

*** Methods for measuring expected returns and portfolio risks.** Markowitz used the concept of expectations in statistical probability theory to create a new concept in finance - expected return [9].

$$E_r = \sum_{i=1}^n R_i \cdot P_i \tag{1}$$

Where, E_r : Expected return;
 R_i : Possible income in case i ;
 P_i : Probability of receiving income;
 n : number of securities.

An indicator characterizing the relative level of risk of a financial asset is the standard deviation of its return from the expected (average for the period) [5]:

$$\sigma_r = \sqrt{D(R)} = \sqrt{\frac{\sum_{i=1}^n (R_i - \bar{R})^2}{n-1}} \tag{2}$$

The measure of dispersion is the standard deviation [5]:

Table 1. Matrix method

Securities	S_1	S_2	...	S_n
S_1	$\sigma_1^2 w_1^2$	$w_1 w_2 \text{cov}(r_1, r_2)$...	$w_1 w_n \text{cov}(r_1, r_n)$
S_2		$\sigma_2^2 w_2^2$		$w_2 w_n \text{cov}(r_2, r_n)$
.	.	.		.
.	.	.		.
S_n	$w_1 w_n \text{cov}(r_1, r_n)$	$w_2 w_n \text{cov}(r_2, r_n)$		$\sigma_n^2 w_n^2$

Source: Hoang Van Quynh & Cao Minh Tien, 2017

*** Selection of an effective portfolio.** For a portfolio of n securities. Each security has a certain expected return and standard deviation, each pair of securities has a certain covariance. To create an effective portfolio, we need to solve two problems.

Objective 1: Minimize risk for a given level of income.

$$\sigma_p^2 = \sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{k=1}^n w_i w_k \text{cov}(r_i, r_k) \rightarrow \min \quad (i \neq k)$$

Restrictions: $\sum_{i=1}^n w_i r_i \geq r^*$, where, r^* - given level of income.

$$\sum_{i=1}^n w_i r_i = 1$$

$$\sigma_i = \sqrt{\sum_{i=1}^n P_i (R_i - E_r)^2} \tag{3}$$

Where, \bar{R} : Average return of the asset for the observed period.

*** Measuring expected returns and portfolio risks.**

The expected return on the investment portfolio is calculated using the following formula [5]:

$$E_{rp} = \sum_{i=1}^n W_i \cdot E(r_i) \tag{4}$$

Where, $E(r_i)$: Expected return of securities i ;

W_i : Share of securities i in the portfolio.

The formula for calculating the variance of a portfolio consisting of n securities is as follows [5]:

$$\sigma_p^2 = \sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{k=1}^n w_i w_k \text{cov}(r_i, r_k) \tag{5}$$

Where, w_i : Share of securities i ;

w_k : Share of securities k ;

$\text{cov}(r_i, r_k)$: Covariant of securities i and k .

When a portfolio contains multiple securities, calculating portfolio variance is relatively complex, requires many pairs of dispersion coefficients between securities, and can easily lead to confusion. To overcome this problem, a matrix method is often used to calculate portfolio variance (Table 1).

$$w_i \geq 0$$

Other restrictions.

Objective 2: Maximize income for a given level of risk.

$$E_{rp} = \sum_{i=1}^n w_i r_i \rightarrow \max$$

Restrictions:

$$\sigma_p^2 = \sum_{i=1}^n w_i^2 \sigma_i^2 +$$

$$\sum_{i=1}^n \sum_{k=1}^n w_i w_k \text{cov}(r_i, r_k) \leq \sigma^{*2} \quad (i \neq k)$$

Where, σ^* - given level of risk.

$$\sum_{i=1}^n w_i = 1$$

$$w_i \geq 0$$

3. ANALYSIS OF THE SECURITIES MARKET OF THE RUSSIAN FEDERATION

3.1. Review of the securities market of the Russian Federation

The Russian stock market has undergone significant development over the past three decades, since the liberalization of the economy in 1991. The dynamic growth of the legal stock market began in the early 2000s as the Russian economy resumed growth. By December 2020, the total market capitalization of Russian companies reached

83.4 trillion rubles, exceeding \$1 trillion at that time. Thanks to this impressive progress, many large Russian corporations and banks have become public companies whose securities are listed on domestic and foreign exchanges [10]. However, the COVID-19 pandemic has slowed economic growth around the world and had a negative impact on financial markets, including in Russia. The data analyzed in this study spans the period from August 2017 to October 2023 and includes 745 data points.

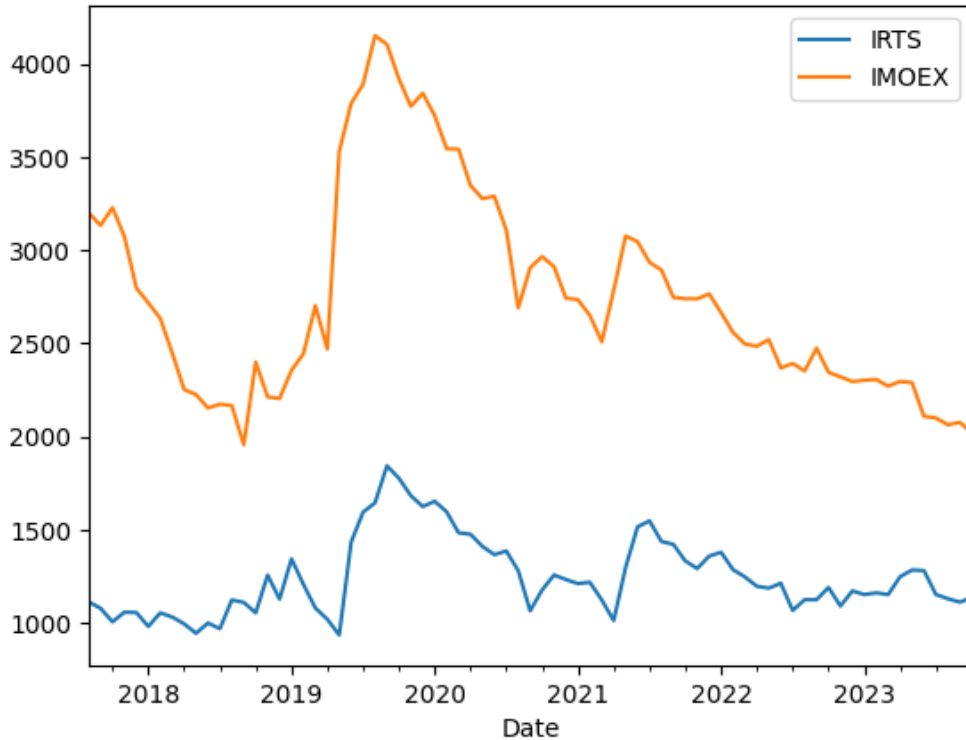


Fig.1. Stock market indices: RTS and MICEX

Source: developed by the authors based on [12]

Analysis of the MICEX and RTS (Russian Trading System) stock indices revealed several key trends. Both indicators tended to fluctuate significantly throughout the study period. The indices reached their peak values in August-September 2021, after which they decreased significantly over the next few months. The most significant drop in the MICEX index occurred in February 2022 - from 3530.38 to 2470.48. The RTS index showed the largest decline in January 2022, falling from 1435.23 to 936.94.

In 2023, the MICEX and RTS indices will recover after previous falls, but remain volatile. The MICEX Index demonstrates more significant fluctuations and relatively higher volatility compared to the RTS Index. In the period

from the end of 2022 to the beginning of 2023, both indices showed positive dynamics, but the MICEX growth rate was lower than the RTS. In addition, the data suggests that there is a correlation between the two indices: they often move in the same direction.

3.2. Statistical analysis of the Russian stock market and factors influencing the Russian stock index

The volatility of the Russian stock market can be analyzed by examining the impact of key macroeconomic variables. The study examined the following important factors: Russia's GDP growth, inflation rate, unemployment rate, key rate, consumer price index, Brent oil price and the US dollar/ruble exchange rate.

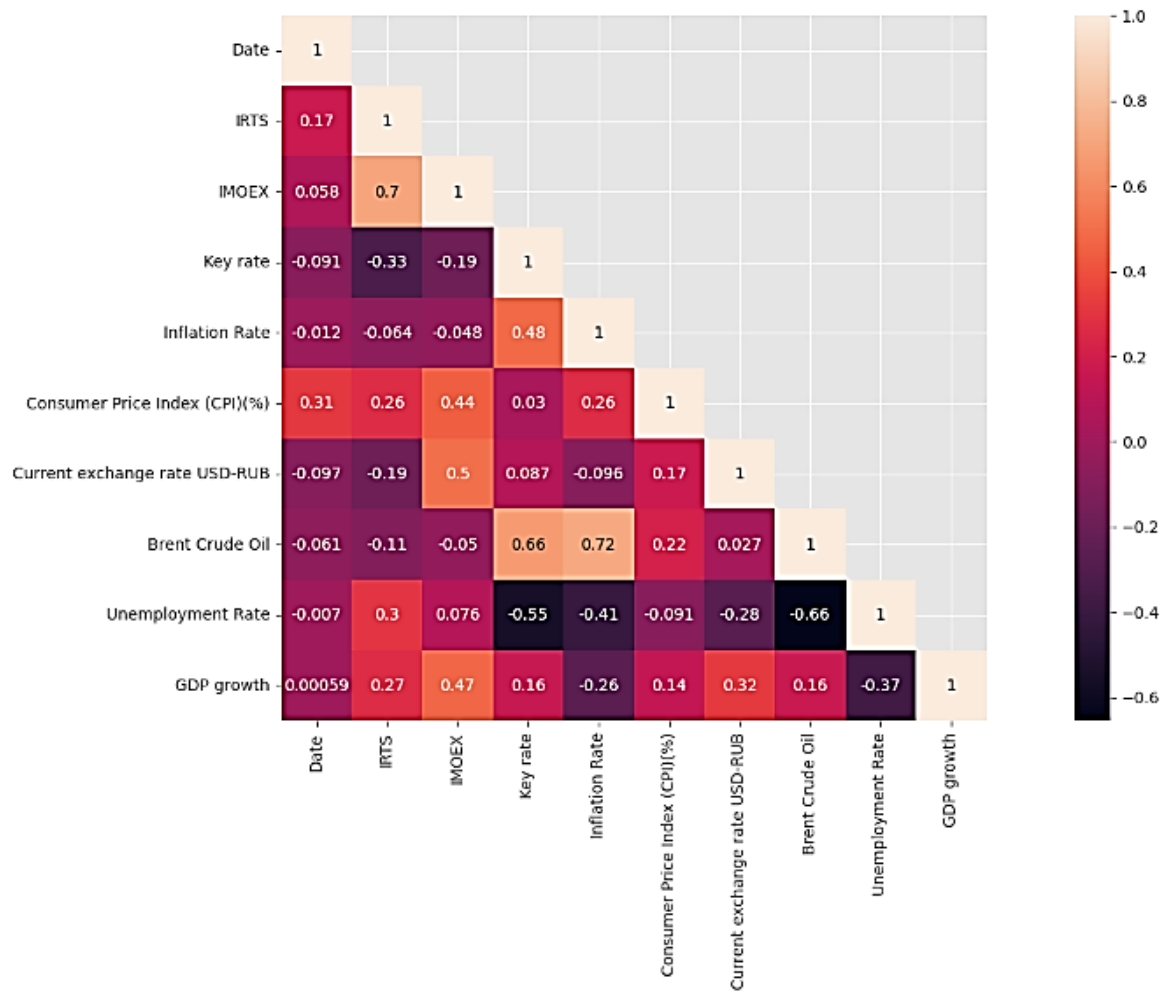


Fig.2. Correlation matrix between the studied variables

Source: Developed by the authors based on [12]

Analysis of the correlation coefficients between these variables and the RTS index allows us to draw several key conclusions:

- RTS Index: There is a strong positive correlation (0.700) between the RTS Index and the MICEX Index, which indicates a close relationship between these two stock indices;

- Dollar/Ruble exchange rate: A positive correlation (0.500) exists between the RTS and the US Dollar/Ruble exchange rate, which suggests that exchange rate fluctuations may have an impact on the RTS;

- GDP growth: RTS shows a positive correlation (0.467) with GDP growth, which means that fluctuations in economic growth rates can affect the RTS;

- Consumer Price Index (CPI): A positive correlation (0.443) between the RTS and the CPI indicates that changes in the CPI may affect the RTS;

- Unemployment Rate: The correlation between the RTS and the unemployment rate is low (0.076), which suggests that the unemployment rate has little impact on the RTS.

Regarding the MICEX index, the analysis showed the following: Unemployment rate: There is a positive correlation (0.299) between the MICEX index and the unemployment rate, which indicates that fluctuations in the unemployment rate can have an impact on the MICEX index.

- GDP growth: The positive correlation (0.266) between the MICEX and GDP growth suggests that changes in economic growth rates can affect the MICEX.

- Consumer Price Index (CPI): The positive correlation (0.264) between the MICEX and the Consumer Price Index suggests that fluctuations in the Consumer Price Index may have an impact on the MICEX.

The article examined various methods for forecasting stock indices, including time series analysis, machine learning and fundamental/technical analysis. The main attention was paid to the use of the ARIMA model for forecasting the RTS and MICEX indices on the Russian market.

After preprocessing the data and splitting the training/test sets in a ratio of 75:25, the auto_arima function was used to determine the optimal ARIMA parameters

(0,1,1). This ARIMA model was then applied to generate forecasts.

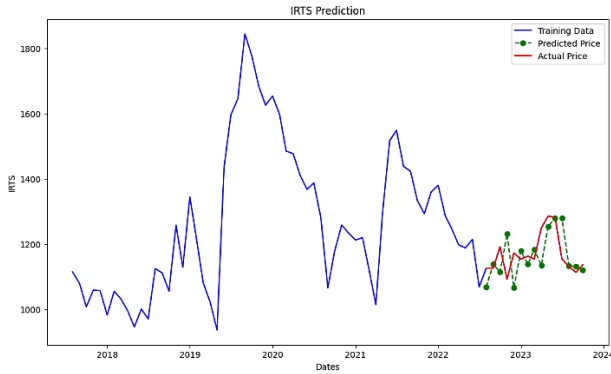


Fig.3. ARIMA model for RTS

Source: Developed by the authors based on [12]

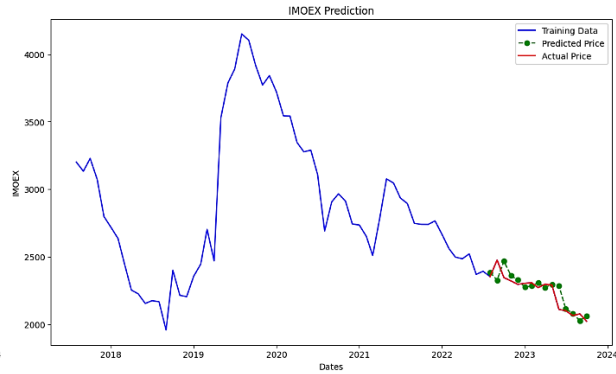


Fig.4. ARIMA model for MICEX

3.3. Optimization of the structure of investment portfolios using the Markowitz model

The analysis begins with an examination of current investment trends among Russian private investors. According to the study, 30% of investors buy stocks, 27% invest in foreign currency, 19% in gold and precious metals,

17% in real estate, 6% in government bonds, antiques and art [10].

Next, the study is devoted to identifying 10 promising Russian stocks in which you can invest, based on the analysis of the IDS Center. The selected stocks included Lukoil, Gazprom Neft, Sovcomflot, Tatneft, Rosneft, Magnit, MMK, Severstal and NLMK [9].

Table 2. List of 10 stocks studied on the Russian stock market

Stock	Companies
RASP	Public corporation Raspadskaya
ROSN	Rosneft
CHMF	JSC Severstal
FLOT	PAO Sovcomflot
TATN	Tatneft
SIBN	Gazprom Neft
LKON	JSC Lukoil
MGNT	JSC Magnit
MAGN	Magnitogorsk Iron and Steel Works
NLMK	Novolipetsk Iron and Steel Works

Source: Finam [9]

Historical monthly stock price data for these 10 securities from October 2020 to October 2023 was obtained from investing.ru. These data were then analyzed using Python libraries such as Pandas, NumPy and Matplotlib [11]. Initial exploratory analysis included visualizing stock price dynamics over time, examining stock price distributions using histograms, and analyzing price volatility using box charts. In addition, a correlation matrix was created to understand the relationship between different stocks.

The analysis showed that the selected stocks do not have a concentrated price range and exhibit significant fluctuations. Additionally, there is no strong correlation between the stocks, suggesting that they may have relatively independent price movements.

The next step in the optimization process is to apply the Markowitz portfolio optimization model to this set of 10 stocks. This model aims to find the optimal combination of stocks that minimizes the overall risk of a portfolio given a

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target expected return. Using historical data and insights gained from exploratory analysis, the Markowitz model can be used to create a well-diversified investment portfolio that matches an investor's risk preferences.

The analysis then continues by selecting the appropriate columns of stock data and calculating the average monthly return for each of the 10 securities.

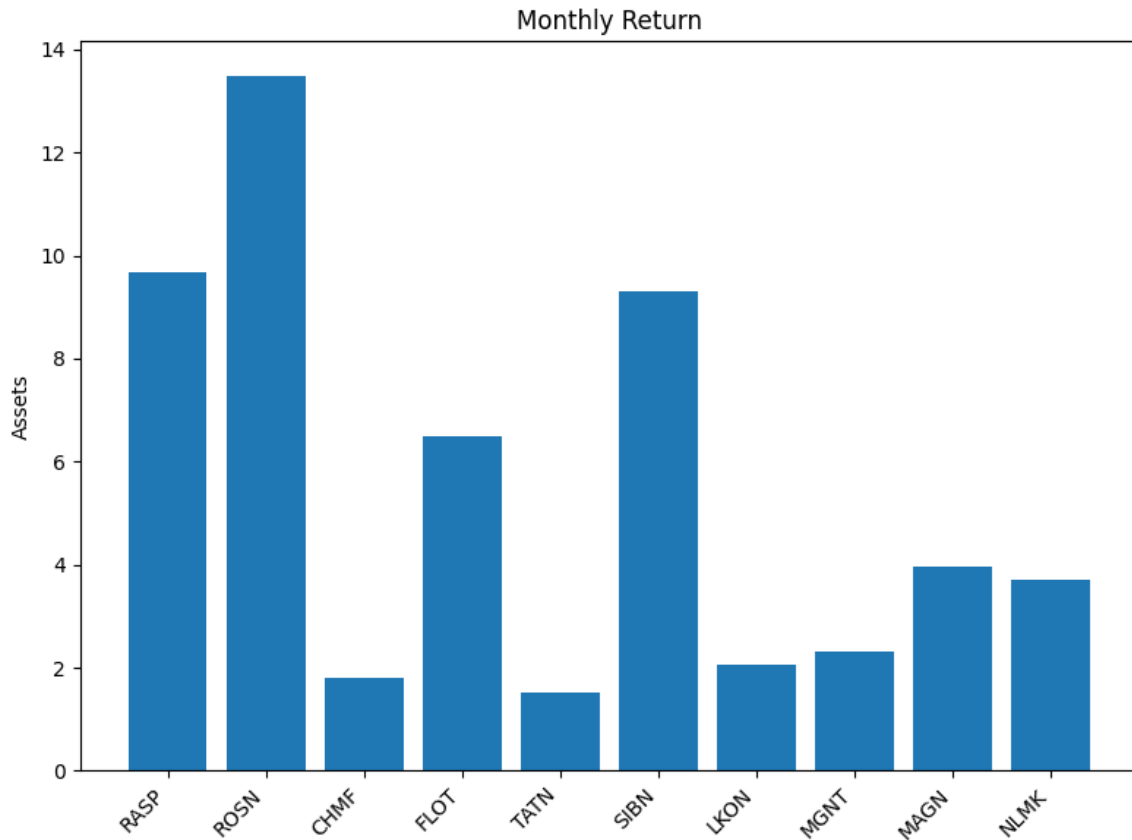


Fig.5. Monthly stock returns

Source: Developed by the authors based on [12]

The results show that Rosneft has the highest average monthly return - 13.48%, and Lukoil and Severstal have the lowest - 2.05% and 1.80%, respectively. The study then focuses on calculating the covariance matrix, which is a square matrix that reflects the volatility and correlation between stocks in a portfolio. The covariance matrix is an essential tool in portfolio analysis and risk management because it helps investors assess the overall risk of a portfolio.

If stocks are highly correlated, the overall risk of the portfolio increases. Conversely, if stocks have low or negative correlations, the portfolio's risk may be reduced. The covariance matrix provides valuable insights. The main diagonal contains volatility levels, with ROSN stock having a high variance of 1079.099, indicating high volatility. There

are positive correlations between pairs of stocks, such as the 198.779 correlation between ROSN and FLOT, which suggests they tend to move together. Negative correlations such as -32.027 between “CHMF” and “ROSN” indicate opposite movements. Diversification can be assessed using a matrix - low or negative correlations, as for SIBN, indicate effective portfolio diversification.

To minimize risk, we use the `scipy.optimize` library and the `minimize` function with the constraint that the total asset weights must be equal to 1. This optimization process allows us to obtain optimal asset weights and the minimum expected portfolio return, which is visualized as a pie chart [5]. We can represent this distribution as the following pie chart.

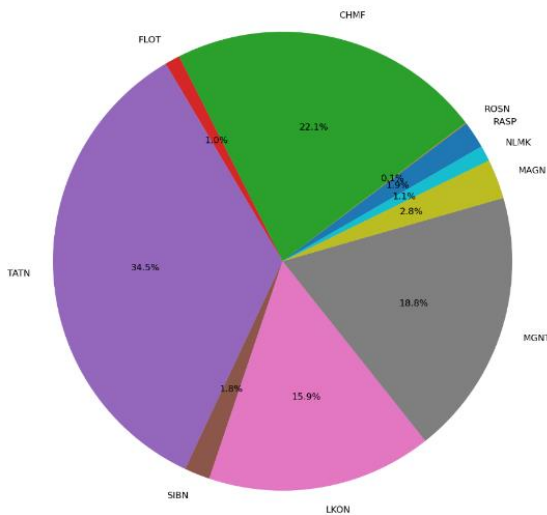


Fig.6. Investment portfolio structure

RASP: 0.0000
 ROSN: 1.0000
 CHMF: 0.0000
 FLOT: 0.0000
 TATN: 0.0000
 SIBN: 0.0000
 LKON: 0.0000
 MGNT: 0.0000
 MAGN: 0.0000
 NLMK: 0.0000

Maximum risk: 32.84964672450105

Fig.7. Weight optimization for maximum profitability

Source: Developed by the authors based on [12]

The minimum return for this portfolio is 2.257. This means that with the above optimal weights we can get a minimum return of 2.257 units. (Fig.6). We also explore the yield maximization objective using a similar optimization approach. As a result, we receive a portfolio consisting entirely of Rosneft shares, with a maximum risk level of 32.85 (Fig. 7).

4. CONCLUSION

This article examines the most important task of managing and optimizing investment portfolios in the Russian financial market. The theoretical foundations of investment portfolios are presented, Markowitz optimization theory is studied, and factors influencing the Russian stock market are analyzed.

An analysis of the main stock indices of the RTS and MICEX revealed significant volatility in 2021-2022, which is affected by the ruble/dollar exchange rate, GDP, inflation and unemployment. A study of 10 Russian securities allowed us to draw several conclusions. To minimize risk, the optimal portfolio was determined: 1.92% RASP, 0.05% ROSN, 22.14% CHMF, 1.05% FLOT, 34.53% TATN, 1.79% SIBN, 15.85% LKON, 18.77% MGNT, 2.79% MAGN, 1.10% NLMK. To maximize returns, it was recommended to invest fully in ROSN.

REFERENCES

1. Davydov A.Yu (2022), Financial markets in the USA and Russia, Russia and America in the XXI Century. URL: <https://elibrary.ru/>.
2. Federal Law "On the Securities Market" dated April 22, 1996 N 39-FZ. SPS "Consultant Plus". URL: <https://www.consultant.ru/> (Date of access: 12/01/2023).

3. Chaweewanchon A., Chaysiri R. (2022), Markowitz Mean-Variance Portfolio Optimization with Predictive Stock Selection Using Machine Learning. International Journal of Financial Studies, 10(3): 64.
4. Harry M. Markowitz (2020), Risk-Return Analysis, McGraw Hill LLC (3): 310
5. Harry M. Markowitz (1952), Portfolio Selection, The Journal of Finance, 7(1): 77-91
6. Hoang Van Quynh & Cao Minh Tien (2017), Investment portfolio management curriculum, Finance Publishing House (Vietnamese version).
7. Nurfadhina Abdul Hali, Ari Yuliati (2020), Markowitz Model Investment Portfolio Optimization: a Review Theory, International Journal of Research in Community Services, 1(3): 14-18.
8. CFA Institute (2020), Portfolio management in practice. Investment management. Wiley, (1): 258
9. Finam, Top 10 investments of December. URL: <https://www.finam.ru/public-ations/item/top-10-investidey-dekabrya-20231203-1300/> (Data of access 03.12.2023)
10. RBC, PSB named the share of Russians with experience. URL: <https://quote.rbc.ru/news/article/6525150d9a79470a05a006bd> (Data of access 10.10.2023)
11. Investing. Finance. URL: <https://investing.ru>
12. Google Colab. URL: <https://colab.google>