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The Algebraic Sets in the General Mathematics and Teaching Them

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ABSTRACT

Most of the pictures in general mathematics are algebraic sets. Indeed, even the first figures taught in class 1 of elementary school are already algebraic sets or part of algebraic sets, such as lines and segments. Therefore, knowing with certainty the properties of algebraic sets is very important for good teaching of high school mathematics, and it is essential to teach them better. To give suggestions and help teachers teach mathematics more effectively, in this report, we will present the Zariski topology, some of their most important properties and the methods to teach algebraic sets.

KEYWORDS: General Mathematics, algebraic sets, Zariski topology, teaching.

INTRODUCTION

The Straight line, plane, circle, hyperbola, parabola, ellipse, sphere, Paraboloid hyperbolic (face saddle), paraboloid elliptic, hyperboloid of one sheet, hyperboloid of two sheets,.... are significant figures in general mathematics (Ministry of Education and Training, 2016). They appear in Geometry, Algebra, Analyze and Probability, and Statistic. This paper will indicate that almost these figures are algebraic sets and present the methods to teach them. We also introduce topological space, Zariski algebraic set, and Zariski topology. Our paper has two parts: 1. Zariski algebraic set and 2. Methods of Teaching Algebraic Set in General Mathematics of Viet Nam. And along with that, the fact that teachers know how to teach the concept of limits of numbers [8], limits of functions [1], and mathematical representations [3] will contribute to improving the quality of teaching.

MATERIALS AND METHODS

For materials, we frequently use additional tools such as computers to design pictures, questionnaires to ask students about their understanding of algebraic sets, field surveys, and referral systems as papers and books for reference. And we use, for methods, the method of analysis, synthesis, survey, and evaluation results for the research.

RESULTS

I. Zariski Algebraic Set

We need the following definitions.

1.1 Polynomial

Let R be the field of a real number and $f(x) := f(x_1, x_2, ..., x_n)$ be a polynomial of n variables or indeterminates $x_1, x_2, ..., x_n$ that means f(x) has the form

$$f = \sum_{r_1 + r_2 + ... + r_n \le d} \lambda_{r_1, r_2, ..., r_n} x_1^{r_1} x_2^{r_2} .... x_n^{r_n}; \ \lambda_{r_1, r_2, ..., r_n} \in R$$

called the coefficients. They are constents. If all coefficients $\lambda_{r_1, r_2, ..., r_n} = 0$, f(x) is called the zero polynomial and denoted by $f(x) = O$.

For exemple, n = 1,

$$f(x) = \lambda_m x^m + \lambda_{m-1} x^{m-1} + ... + \lambda_1 x + \lambda_0 ;$$

n = 2, $g(x, y) = ax^2 + bx - y + d;\ a, b, c \in R$. Then f(x) and g(x, y) are polynomals.

Denote by

$R[x] := R[x_1, x_2, ..., x_n] = \{f(x) = f(x_1, x_2, ..., x_n);\ f \text{ is a polynomal}\}$

. It is called the polynomial ring (Nguyen Huynh Phan, 2012; Justin R. Smith, 2014; Ngo Viet Trung, 2012).

The n-dimensional affine space over R is defined as the set R^n. The elements R^n are called points. R^1 is called the affine line and R^2 is called the affine plane.

Let $f \in R[x_1, x_2, ..., x_n]$. A point $P = (a_1, a_2, ..., a_n)$ is a root or zero of the polynomial f if and only $f(P) = f(a_1, a_2, ..., a_n) = 0$

1.2. Algebraic Set

A subset $V \subseteq R^n$ is called an affine algebraic set or an algebraic set, or Zariski algebraic set if and only if there is a set of the polynomial $S \subseteq R[x_1, x_2, ..., x_n]$. Such that $V = V(S) = \left\{P \in R^n;\ f(P) = 0 \text{ for all } f \in S\right\}$. V is called the root set of the set S (John L. Kelley, 1972; Nguyen Huynh Phan, 2012; Ngo Viet Trung, 2012).

1.2.1.Proposition: *The following sets are the algebraic sets:*

1/ The empty set $\varnothing$*;*

2/ R^n*;*

3/ Any single point P in R^n *is an algebraic set.*

Proof: Because the empty set $\varnothing$ is the root set of the polynomial $x^2 + 1 = 0$ and R^n is the root set of the zero polynomial O; The point $P = (a_1, a_2, ..., a_n)$ is the root set of n polynomials of the from $f_i(x) = x_i - a_i; i = 1, 2, ..., n$.

1.2.2.Theorem: *The collection T of all Zariski algebraic sets establishes a topology* R^n. *That means, this collection T satisfies three conditions following (Doan Quynh and Hoang Xuan Sinh, 1987; John L. Kelley, 1972):*

(1) $\varnothing, R^n \in T$

(2) $A_1, A_2 \in T \Rightarrow A_1 \cup A_2 \in T$;

(3) $A_i \in T (i \in I) \Rightarrow \bigcap_{i \in I} A_i \in T$.

Proof: By The Proposition 1.2.1. $\varnothing, R^n$ are Zariski algebraic sets, then $\varnothing, R^n \in T$. Suppose $A_1, A_2 \in T$. By the definition of Zariski algebraic set, exist $S_1, S_2 \subseteq R[x_1, x_2, ..., x_n]$ such that A_1, A_2 are alternately the root sets of S_1, S_2. Denote by $S_1S_2 = \{g_1g_2;\ g_1 \in S_1 \text{ and } g_2 \in S_2\}$. Then $A_1 \cup A_2$ is the root set of S_1S_2. Therefore $A_1 \cup A_2$ a Zariski algebraic set. Final, suppose A_i is the root set of $S_i, i \in I$. Then $\bigcap_{i \in I} A_i$ is a root set of $\bigcup_{i \in I} S_i$, hence $\bigcap_{i \in I} A_i$ is a Zariski algebraic set.

1.2.3.Theorem: *The Straight line, plane, circle, hyperbola, parabola, ellipse, sphere, paraboloid hyperbolic (face saddle), paraboloid elliptic, hyperboloid of one sheet, hyperboloid of two sheets are the algebraic sets.*

Proof: Denote by :

$f_1(x, y) = ax + by + c;\ a, b, c \in R;\ a^2 + b^2 \neq 0;$

$f_2(x, y, y) = ax + by + cz + d;\ a, b, c, d \in R;\ a^2 + b^2 + c^2 \neq 0;$

$f_3(x, y) = (x - a)^2 + (x - b)^2 - r^2;\ a, b, r \in R;\ r > 0;$

$f_4(x, y) = \frac{x^2}{a^2} - \frac{x^2}{b^2} - 1;\ a, b \in R;\ a, b > 0$

$f_5(x, y) = ax^2 + bx - y + d;\ a, b, c \in R;$

$f_6(x, y) = \frac{x^2}{a^2} + \frac{x^2}{b^2} - 1;\ a, b \in R;\ a, b > 0$

$f_7(x, y, z) = (x - a)^2 + (x - b)^2 + (z - c)^2 - r^2;\ a, b, c, r \in R;\ r > 0;$

$f_8(x, y, z) = \frac{x^2}{a^2} - \frac{y^2}{b^2} - 2z;\ a, b \in R;\ a, b > 0;$

$f_9(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 2z;\ a, b \in R;\ a, b > 0$

$f_{10}(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} - 1;\ a, b, c \in R;\ a, b, c > 0;$

$f_{11}(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} + 1;\ a, b, c \in R;\ a, b, c > 0.$

Then, the Straight line, plane, circle, hyperbola, parabola, ellipse, sphere, Paraboloid hyperbolic (face saddle), paraboloid elliptic, hyperboloid of one sheet, hyperboloid of two sheets are alternately the root set of the polynomials f_i, i = 1, 2, …., 10 and 11.

1.2.4.Theorem: *A subset* $V \subseteq R$ *is a Zariski algebraic set if and only if V is the empty set or V =* R *or V is a finite set.*

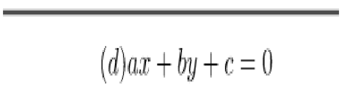
Proof: Because all polynomial f(x) of one indeterminate has only finite the root. Hence if V is a Zariski algebraic set, that means V is the root set of polynomials; therefore, if V is not $\varnothing$ or R^n, then V is a finite set.

II. METHODS OF TEACHING ALGEBRAIC SET IN GENERAL MATHEMATICS OF VIET NAM

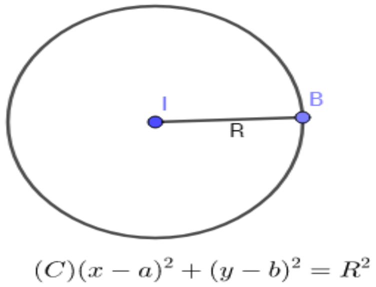
2.1 Use geometry to teach algebra set

We use methods drawing the figures and intersecting an asymptote to teach an algebraic set.

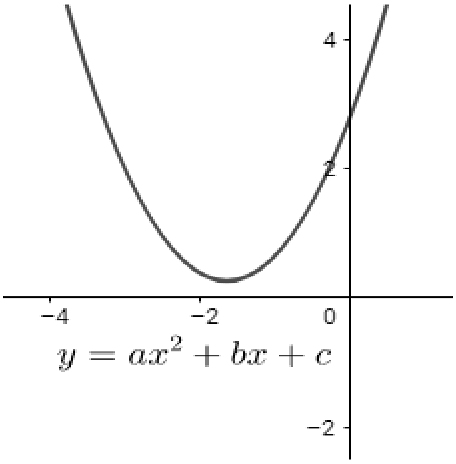
2.1.1. Draw the figures: Drawing the figures, for example, the figures given in Theorem 1.2.3, we have as following



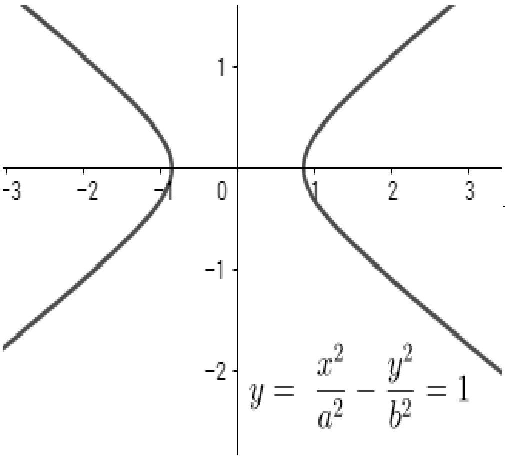
$(d) ax + by + c = 0$



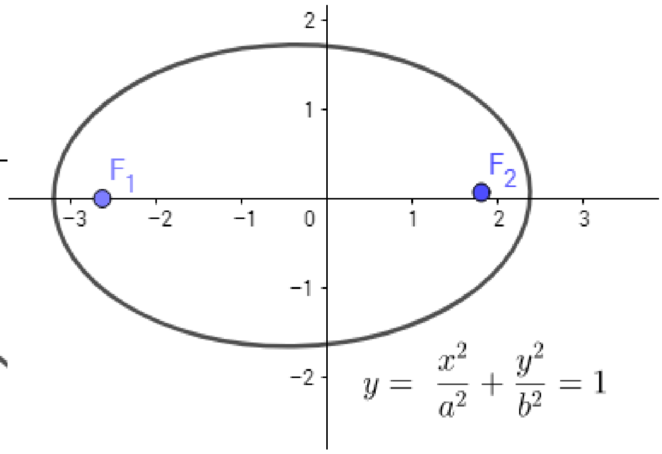
I
B
R
$(C)(x-a)^2 + (y-b)^2 = R^2$



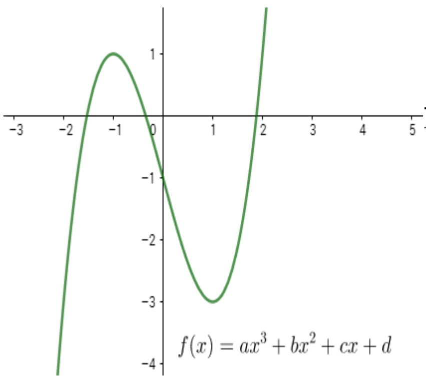
$y = ax^2 + bx + c$



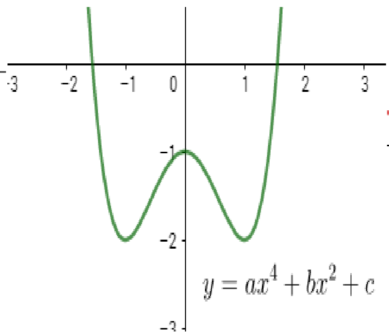
$y = \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



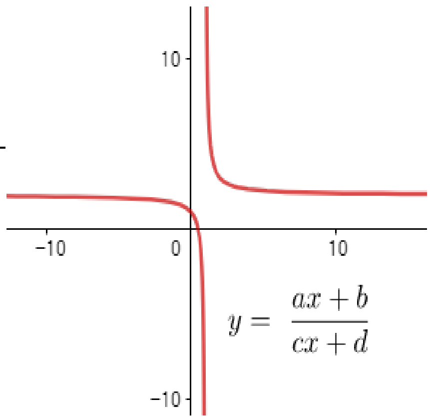
F_1
F_2
$y = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



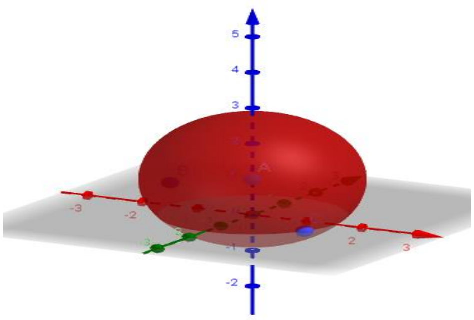
$f(x) = ax^3 + bx^2 + cx + d$

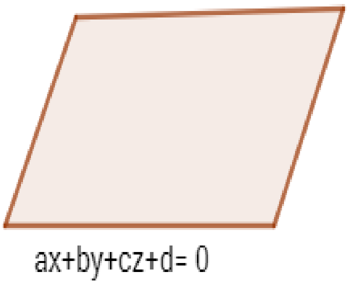


$y = ax^4 + bx^2 + c$

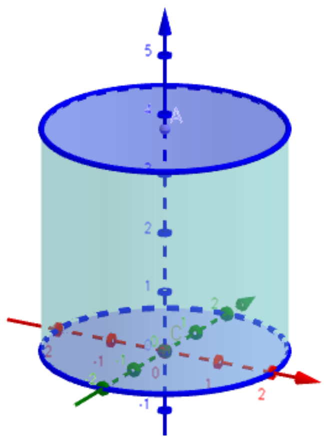


$y = \frac{ax+b}{cx+d}$





ax+by+cz+d= 0



2.1.2. The method of intersecting

In the Vietnamese mathematics textbook, the algebraic set appears and occupies positions in the geometric program. Many mathematical problems of correspondence among the figures refer to the algebraic set.

Problem 1: In the plane, consider the relative position between the two lines

$$d_1: a_1x + b_1y + c_1 = 0\ ,\ d_2: a_2x + b_2y + c_2 = 0$$

Solution: We have $f_1(x,y)$ and $f_2(x,y)$ are the solution of two variable polynomials x, y

$$\begin{cases} f_1(x,y) = a_1x + b_1y + c_1 \\ f_2(x,y) = a_2x + b_2y + c_2 \end{cases} (*)$$

consider the relative position between the two lines that mean, we solution of the polynomial of two variables x, y

- If (*) has one solution $\Leftrightarrow$ d_1 intersecting d_2: $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0 \Leftrightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

- If (*) hasn't a solution $\Leftrightarrow d_1 // d_2$

$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0$ and $\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \neq 0$ or $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0$ and $\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix} \neq 0$

Therefore : $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

- If (*) has infinitely many solutions $d_1 \equiv d_2$

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix} = 0 \Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Example 1: $d_2 : x + (m-1)y + m - 3 = 0$

Find m to two lines: intersection, coincidental, parallel

We have (I) $\begin{cases} d_1 : (2m-1)x + y + 3 = 0 \\ d_2 : x + (m-1)y - 3 = 0 \end{cases}$

- d_1 : intersecting d_2 if (I) has one solution $\frac{2m-1}{1} \neq \frac{1}{m-1} \Leftrightarrow m \neq 0$ và $m \neq \frac{3}{2}$

- $d_1 \equiv d_2$ if (I) has infinitely many solutions $\frac{2m-1}{1} = \frac{1}{m-1} = \frac{3}{m-3} \Leftrightarrow m = 0$

- $d_1 // d_2$ if (I) hasn't the solution $\frac{2m-1}{1} = \frac{1}{m-1} \neq \frac{3}{m-3} \Leftrightarrow m = \frac{3}{2}$

Similarly: In space, have two plane

$$(P_1): A_1x + B_1y + C_1z + D_1 = 0$$
$$(P_2): A_2x + B_2y + C_2z + D_2 = 0$$

- (P_1) intersecting (P_2) $\Leftrightarrow A_1 : B_1 : C_1 \neq A_2 : B_2 : C_2$

- $(P_1) // (P_2) \Leftrightarrow \frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} \neq \frac{D_1}{D_2}$

- $(P_1) \equiv (P_2) \Leftrightarrow \frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = \frac{D_1}{D_2}$

Problem 2: Consider the relative position between a line $ax + by + c = 0$ and a circle $(x-a)^2 + (y-b)^2 = R^2$. That means we have a solution to the polynomial of two variables x, y.

Example 2: Find m to consider the relationship (d): $3x - y + m = 0$ and $(c) x^2 + y^2 - 4x + 6y + 3 = 0$.

We have (I) $\begin{cases} x^2 + y^2 - 4x + 6y + 3 = 0 \\ 3x - y + m = 0 \end{cases}$

- d intersecting (C) if (I) has two solutions : $-19 < m < 1$
- d contacts (c) if (I) has one solution $m = 1, m = -19$
- d and (c) are not intersection if(I) hasn't the solution: $m < -19$ hay $m > 1$

Similarly: In the space, consider the relationship

$d : \begin{cases} x = 2t \\ y = 1 - t \\ z = 3 + t \end{cases}$ and (P) $x + y + z - 10 = 0$.

That means we find solution t: $2t + 1 - t + 3 - 10 = 0$ $\Leftrightarrow t = 3$. Therefore, (d) and (P) have one point common.

Problem 3: Consider the relationship between the functions.

Example 3: Find m to (d): $y = -x + m$ intersecting the function (C) $y = \frac{x}{x-1}$ at two distinct points

We find the intersection: $-x + m = \frac{x}{x-1} \Leftrightarrow$

$f(x) = x^2 - (m-2)x - m = 0$ ($x \neq -1$)

(d) intersecting the function (C) at two distinct points $\begin{cases} \Delta = (m+2)^2 > 0 \\ f(-1) = -1 \neq 0 \end{cases} \Leftrightarrow m \neq -2$

Example 4: Find m to $y = -1$ intersecting at four distinct points and the x- coordinate of the points is less than 2.

We have: $x^4 - (3m+2)x^2 + 3m = -1$

$\Leftrightarrow x^4 - (3m+2)x^2 + 3m + 1 = 0$

$\Leftrightarrow \begin{cases} x = 1, x = -1 \\ x^2 = 3m + 1 \quad (*) \end{cases}$

$y = -1$ intersecting at four distinct points, the x- coordinate of the points is less than 2 if and only if equations (*) have two distinct solutions $x < 2$.

$$\Leftrightarrow \begin{cases} 0 < 3m+1 < 4 \\ 3m+1 \neq 1 \end{cases} \Leftrightarrow \begin{cases} -\frac{1}{3} < m < 1 \\ m \neq 0 \end{cases}$$

Example 5: Find m to a line $y = -x + 3$ interesting (c) at three distinct points.

We have: $x^3 - 3(m+1)x^2 + mx + 3 = -x + 3$

$$\Leftrightarrow x[x^2 - 3(m+1)x + m + 1] = 0$$

$$\Leftrightarrow \begin{cases} x = 0 \\ f(x) = x^2 - 3(m+1)x + m + 1 = 0 \end{cases}$$

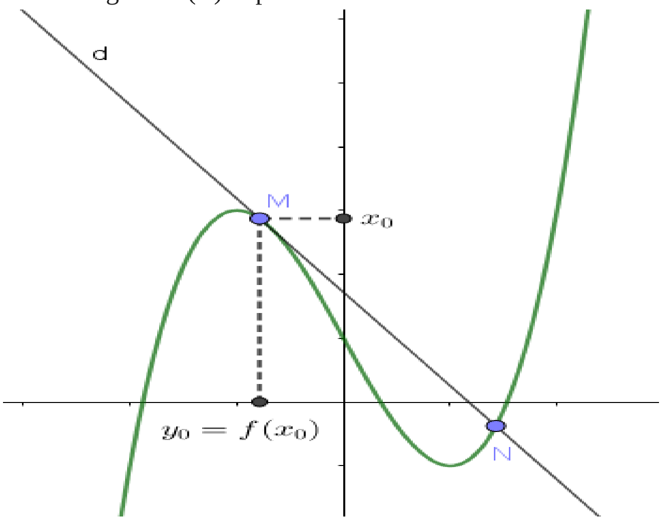
a line $y = -x + 3$ interesting (c) $y = x^3 - 3(m+1)x^2 + mx + 3$ at three distinct points if f(x) has two distinct points and x ≠ 0

$$\begin{cases} \Delta = 9(m+1)^2 - 4(m+1) > 0 \\ f(0) = m + 1 \neq 0 \end{cases} \Leftrightarrow \left[\begin{array}{l} m < -1 \\ m > -\frac{5}{9} \end{array} \right.$$

Comment: Any problem of the relationship between lines in general mathematics can be used algebraically to solve. The distribution of algebraic sets will be an algebraic set.
The solution to these problems is to use algebra. It then uses algebraic sets to study any geometry in general mathematics.

2.2. Tangent function:

2.2.1. Definition: Assume that the function y = f (x) has a graph that is denoted by (C), a line tangent to (C) at the point called tangent of (C) at point M



In this definition, the tangent is essentially the intersection of two sets of algebraic terms "d in contact with (C)": d in contact with (C) at point M and cut (C) at point N.

Point $M(x_0, y_0)$ is called contact points (tangent points) of tangents and graphs. Since point M is a graph of the function y = f (x) so $y_0 = f(x_0)$.

In general mathematics, we assume that the tangent of tangents at the principal point is the derivative of the function y = f (x) at the point x_0. So we get the tangential equation:

$$y - y_0 = f'(x)(x - x_0).$$

2.2 Asymptote

2.2.1.Vertical asymptotes

The line $(d) x = x_0$ is a *vertical asymptote* of the graph of the function y = f(x) if at least one of the following statements is true

$\lim_{x \to x_0^-} f(x) = +\infty$ or $\lim_{x \to x_0^+} f(x) = +\infty$ or $\lim_{x \to x_0^-} f(x) = -\infty$ or $\lim_{x \to x_0^+} f(x) = -\infty$.

2.2.2.Horizontal asymptotes

The line (d) $(d) y = y_0$ is a horizontal asymptote of the graph of the function y = f(x) if $\lim_{x \to +\infty} f(x) = y_0$ or $\lim_{x \to -\infty} f(x) = y_0$.

2.2.3.Oblique asymptotes

The line (d) $y = \text{ax+ b}$ ($a \neq 0$) is an oblique asymptote of the graph of the function

$y = f(x)$ if $\lim_{x \to +\infty} [f(x) - (\text{ax+b})] = 0$ or $\lim_{x \to -\infty} [f(x) - (\text{ax+b})] = 0$.

CONCLUSION

All pictures and more than half of the problems of general mathematics are algebraic sets, although the algebraic set is not taught explicitly, so the teacher needs knowledge of algebraic set and algebraic geometry to teach general mathematics, they will be deeper.

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