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Fractal Structure and Fractal Measurement of Pulmonary Vascular Systems

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ABSTRACT

This article is devoted to determining the fractal structure and fractal size of organs. There is a detailed description of the various mathematical methods for determining the size of fractal organs, and an analysis of errors in determining the fractal size of organs. In the article, the fractal structure, fractal size, properties of human organs were determined using the Mandelbrot-Richardson scale (or cell method). The fractal structure of the human lung was also studied by comparing the fractal structure of tree branches. In particular, tree branches, vascular systems in the human retina, and fractal measurements of the lungs were calculated. In determining the fractal scale, changes in human body parts were not taken into account. Most articles have used fractal measurement only in relation to geometric shapes. In this article, the fractal structure of human organisms is studied on the basis of mathematical formulas and special methods are used to calculate fractal dimensions, as well as the results of an appropriate number of experiments. In addition, based on these measurements, fractal measurements of the pulmonary vascular system of patients have been evaluated in several studies, widely used to describe vascular networks in various diseases, and data on their practical application have been provided.

KEYWORDS: Fractal, fractal measurement, Mandelbrot-Richadson measurement, pulmonary vascular systems.

1. INTRODUCTION

Fractals are used in computer systems, telecommunications, industrial production and many other fields of science and technology [1-7]. One of the most interesting areas is the application of these fractals in medicine. The fractal organs of the human body are the entire respiratory tract, vascular system, lymphatic vessels, liver and bile ducts, as well as the nervous system [8], the fractal dimensions of which are determined in different ways.

The vascular systems in the human retina are statistically similar and fractal [9]. It corresponds to an irregular but limited growth process and can affect the embryological development of the vascular system. In addition, the human bronchial and respiratory tracts also have a fractal structure.

At present, it is important to determine the fractal size of organs with a fractal structure in the human body, on the basis of which to help to predict and treat various diseases in humans [10]. Therefore, this article focuses on determining the fractal dimensions of human organs. Methods for determining the fractal size of tree branches were used to find fractal measurements of human organs.

The fractal size of the lungs was determined by R.V. Genny using the vector method, [9] and in this paper the fractal size of the lungs was determined using the grid method, also known as the Mandelbrot-Richardson gauge method or the grid method. In determining the fractal dimensions of the vascular systems in the human lung and retina, the location of the vessels was studied by comparing them to tree branches. It was therefore first found in the fractal dimensions of tree branches and then applied to the human limb, which has a fractal structure.

2. THE MAIN FINDINGS AND RESULTS

Fractal model of world cognition, which is inextricably linked with the processes of self-organization. This concept is derived by us from several existing problems of brain research and the above postulates. Those areas of knowledge that form the synergetics of fractal systems are genetically revealed. These sources are reflected as the history of individual sciences: mathematics, physiology of the senses, and psychology [12,13].

The fractal model is based on a mathematical basis, and in philosophical research, the results of mathematics and physics very often serve as proof of theoretical conclusions. It is proved that the basis of nonlinear structures of the external world is a geometric self-similar fractal [14].

The study of natural fractal structures enables us to gain a deeper understanding of the processes of self-organization and development of nonlinear systems. Most of the surfaces of objects in the world around us are fractal and are jagged and rough. Although for a long time only polished, geometrically smoothed surfaces of objects have been studied in science. The mathematician B. Mandelbrot [1] introduces the concept of a fractal, which more adequately reflects the morphology of formless bodies, taking into account the peculiarities of the structure of the world. He describes a wide variety of objects, ranging from the coastline, metal alloys, cloud silhouettes, and to many other natural and artificial objects. In the study, a fractal is understood as a certain formation that has the property of self-similarity and self-affinity. It has a regular geometric structure, where each fragment of the fractal repeats the entire structural structure as a whole.

Based on the similarity of self-organization processes, a hierarchy of levels of interconnected systems is built: the physical world, neural networks of the human brain and the psychological system. It is these levels that define the geometrical order in the world. According to scientists, the fractal picture of the world, reflected in the human brain, is self-organizing in neural networks at a different, higher level and will also have a fractal structure, that is, a structure that is self-similar and self-affine to the inner world. Consequently, the newly formed neural connections will be able to set the structure of human interaction, expressed in the development of the psychological system. Taking into account the conditions that the formation of a fractal occurs according to a single mathematical formula in which the parameters are strictly defined, and the forming structure is harmonious, then all three levels: the structure of the external world, the structure of neural networks of the human brain and the structure of the interaction of individuals - will be determined by one self-similar fractal. It is the creation of such a fractal geometry that corresponds to the self-organization of the systems of the integral real world. We extend the fractal property of systems to the psychological world, interpreting them from the broadest positions of development and unity of the world, which differ from each other in space-time scales. We synergistically combine the selected levels of the fractal picture of the world into a dynamic nonlinear system with many different dimension values. We get a holistic structure and characterize it as a multifractal. We introduce a refinement, according to which a similarity is observed between a fractal and an attractor. At the same time, many phenomena in inanimate and living nature are described in terms of sinks, cycles, attractors and strange attractors. Such phenomena should be revealed on the basis of a single fractal structure. When the dimension is more than two, but less than three, the properties of the strange attractor of Lorentz appear [15]. In translation from English, a strange attractor means "attractor". It is understood as a set of trajectories in the phase space. It also attracts various trajectories that lie in the vicinity of the attractor. Thus, the fractal structure acquires a certain order, and the fractal itself becomes a very convenient model for studying self-organization and the development of nonlinear systems [16, 17]. So, the dialectical concept of development is confirmed by synergetics and the theory of fractals. It relies on physical thermodynamics and mathematical set theory, systemic and structural approaches that interpret the processes of development of inanimate and living nature using nonlinear methods of understanding the world, due to the universality of self-organizing processes of various levels.

The human brain is also represented as a fractal hologram. The brain reflects the world around it. It is part of a broader system. Emphasis is placed on the structural and functional mechanisms of the brain. The human brain, like many other systems, operates in a nonlinear, chaotic mode [18]. The processes of self-organization of neural structures are constantly going on in it.

Nonlinear dynamics is the basis of little-studied phenomena. The functioning of the human brain is presented as an integral non-linear system of the external and internal world. To understand the self-organization of such nonlinear systems, a new synergetic approach is needed, because the study of individual levels of its constituent systems by various separate methods does not give a complete, all-embracing picture of the whole. Although rather complex measurements of the electrical and magnetic fields of the brain are used, which are created in the process of interaction of many neurons. It is important to note that, in contrast to the traditional approach, the synergistic approach operates not with individual cells, but with a neural network.

One of the directions of modeling neurosystems is a neurocomputer, which functions on the basis of a self-organization model and, accordingly, makes it possible to study the union of neurons into a system with certain behavior properties. It should be noted, however, that there are fundamental differences between brain activity and the machine that simulates it. The parallel processes of a functioning brain do not correspond to the sequential processes of a PC. If in the computer algorithm the program is rigidly specified, then in the new interpretation the neural network is self-organizing [19].

Fractal dimension of organs and organisms. Fractals not only surround us, they are also inside us and many animals and plants, since many organs of the human and animal body, as well as plants, have fractal properties. Using the capabilities of fractal structures, nature designed the human body extremely effectively.

The most thoroughly studied is the fractal structure of the airways, through which air enters the lungs.

The lungs are vital organs responsible for the exchange of oxygen and carbon dioxide in the human body and perform the respiratory function. The diagram of the lungs includes three important structural elements: bronchi, bronchioles and pulmonary alveoli.

The framework of the lungs is a branched bronchial system. Each lung is made up of many structural units. Each slice has a pyramidal shape with an average size of 15x25 mm. The bronchus enters the apex of the lung lobule, the branches of which are called small bronchioles. In total, each bronchus is divided into 5-20 bronchioles. At the ends of the bronchioles there are special formations - acini, consisting of several dozen alveolar branches covered with many alveoli. The most important structural elements of the lungs are the alveoli, on which the normal exchange of oxygen and carbon dioxide in the body depends. Pulmonary alveoli are small vesicles with very thin walls, braided by a dense network of capillaries. Thanks to microscopic alveoli, the average diameter of which does not exceed 0.3 mm, the area of the respiratory surface of the lungs increases to 80 square meters. They provide a large area for gas exchange and continuously supply oxygen to the blood vessels. In the course of gas exchange, oxygen and carbon dioxide penetrate through the thin walls of the alveoli into the blood, where they "meet" with erythrocytes [20]. Thus, the lungs are an example of how a large area is "squeezed in" into a rather small space.

The bronchi and bronchioles of the lung form a "tree" with numerous branches. A quantitative analysis of the branching of the airways showed that it has fractal geometry.

Methods for calculating fractal dimension. In practical problems, the calculation of the fractal dimension is most often carried out on the basis of the cubes method, the coating method, the local dispersion method, the prism method, and a number of others. However, even when processing the same image using different methods, the results are often different from each other.

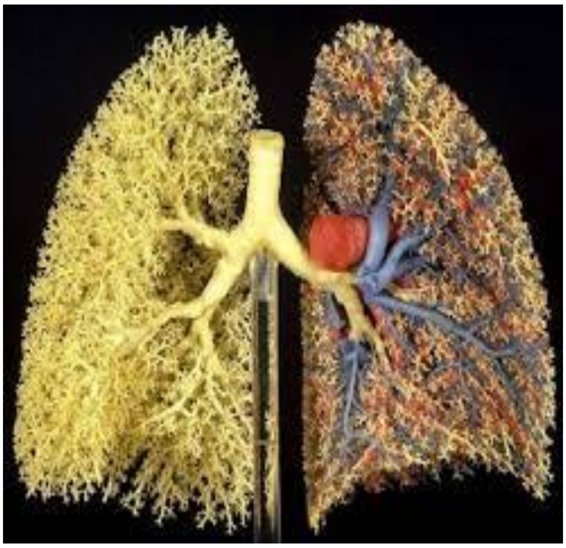

Figure 1. The structure of the respiratory tract

When calculating the fractal dimension in practice, one should choose an appropriate algorithm based on considerations of computational accuracy, speed, and system resources [18].

3. MANDELBROT-RICHARDSON MEASUREMENT OR GRID METHOD.

Complex geometric objects with fractal structures can be described and studied by mathematical methods. The analysis of the placement density of tree branches can be viewed as a quantitative determination of the filling of this gap. Thus, the closer the value of the fractal dimension of two-dimensional branched horns to two, the more effectively the tree fills the space, because the upper limit of the fractal dimension corresponds to the topological dimension. The fact is that the images of fractal geometric objects are usually always considered to be in the plane [8,9]. We can see how much area fractal images occupy in the plane. To do this, divide the plane into cells, the size of which is denoted by a, and calculate how many cells intersect the fractal images.

Determination of the fractal scale of tree branches using the Mandelbrot-Richardson scale:

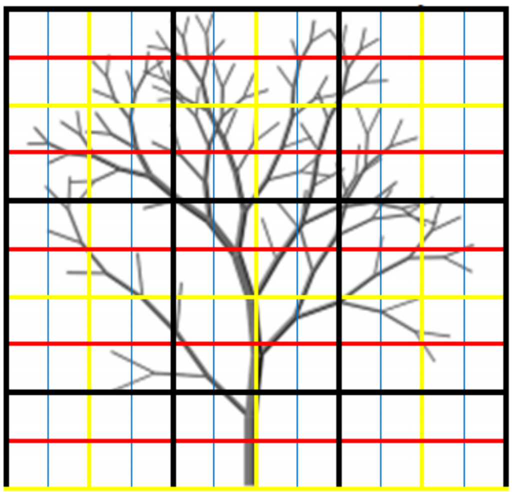

Figure 2. Three different-sized squares were drawn on the horns of Darat. From this it was determined that: a is the size of the cell, conditionally a = 48 mm, the number of cells in the drawing is corresponding, the number of cells in black is $N_1 = 7$, the number of cells in yellow is $N_2 = 22$, the number of cells in blue is $N_3 = 73$.

The above N and a are related to the Mandelbrot-Richardson formula: That is,

$$N = C \times a^{-D} \qquad (1)$$

where, $D -$ is the fractal dimension, $C -$ is the size characteristic of fractal geometry. Fractal measurement indicates the degree to which a flat surface is filled with a fractal drawing [21].

Table 1. The results of measuring the number of cells in which the lines.

The size of the cell a	9	16	48
Number of cells N	73	22	7
$y = lnN$	4,2904	3,0910	1,9459
$x = lna$	2,1972	2,7726	3,8712

Analysis of the placement density of tree branches is done by calculating their fractal dimensions. We will look at this in more detail in the example in Figure 2. The results of measuring the number of cells in which the lines of the drawing are located depend on the cell size are given in Table 1.

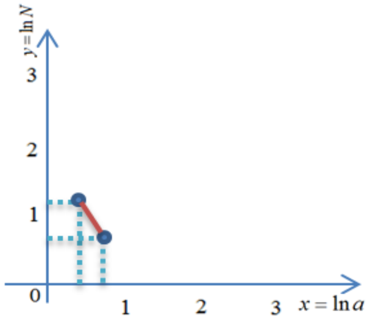

Figure 3. Creating a straight line

Based on these, logarithmic values were also calculated. As can be seen from the graph in Figure 3, a straight line is formed between the two points. That is,

$$y = -D \cdot x + c \qquad (2)$$

This is the fractal dimension sought in formula $D -$ Now we add all x and y in Table 1:

$$\sum_{i=1}^{n} y_i = n \cdot c - D \cdot \sum_{i=1}^{n} x_i \qquad (3)$$

(3) is multiplied by both sides of the formula:

$$\sum_{i=1}^{n} x_i y_i = c \cdot \sum_{i=1}^{n} x_i - D \cdot \sum_{i=1}^{n} x_i^2 \qquad (4)$$

The formula for finding the fractal scale using the Mandelbrot-Richardson scale.

$$D = \left(\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i - n\sum_{i=1}^{n} x_i y_i\right) \Big/ \left(n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right) \qquad (5)$$

If we put the data given in Table 1 above in (5), the fractal size of the tree branches is determined. That is,

$$D = \left(\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i - n\sum_{i=1}^{n} x_i y_i\right) \Big/ \left(n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right)$$
$$= 1{,}3531 \qquad (6)$$

Determination of the fractal scale of the complex structure of tree branches using the Mandelbrot-Richardson scale:

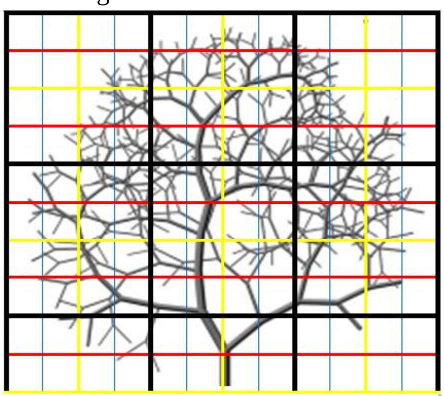

Figure 4. On the horns of the tree are drawn squares of three different sizes. From this it was determined that: a is the size of the cell, conditionally a = 48 mm, the number of cells in the drawing is corresponding, the number of cells in black is $N_1 = 8$, the number of cells in yellow is $N_2 = 27$, the number of cells in blue is $N_3 = 89$

In Fig. 4, the fractal dimension in the complex case of tree horns was determined (it has a more complex structure than in Fig. 2).

Table 2. The number of cells in which the tree branches are located

The size of the cell a	9	16	48
Number of cells N	89	27	8
$y = lnN$	4,4886	3,2958	2,0794
$x = lna$	2,1972	2,7726	3,8712

Based on the above data, the fractal dimension in the complex case of tree branches is given by Equation 5 as follows.

$$D = \left(\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i - n\sum_{i=1}^{n} x_i y_i\right) \Big/ \left(n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right) =$$
$$= 1{,}3952 \qquad (7)$$

Hence, instead of concluding from the results of 6 and 7, it can be said that the higher the surface coverage of a given shape, the more accurate its fractal dimension.

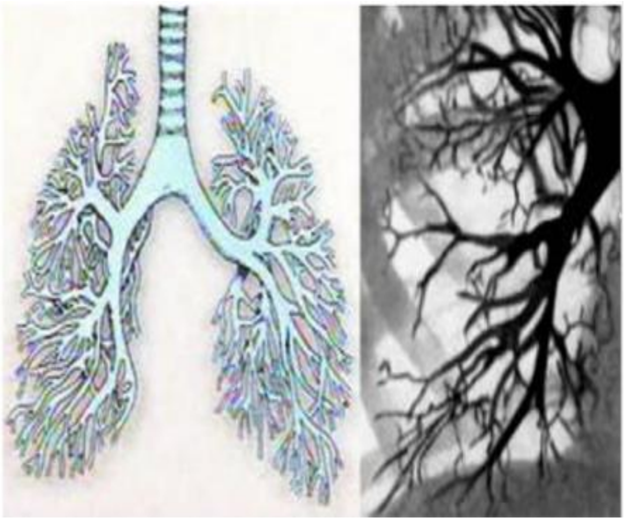

Figure 5. This picture shows that the human lung has a tree-like structure.

Evaluation of the fractal scale can also be used to characterize the human retina, various tumor formations [17], as well as to analyze the three-dimensional arterial tree of the human lung obtained using computed tomography data. It appears that, at the very least, the arterial system of the lung consists of a combination of two components: a capillary network that uniformly fills the cavity, and a scattered fractal structure of large vessels.

Using the above method, we determine the fractal size of the human lung:

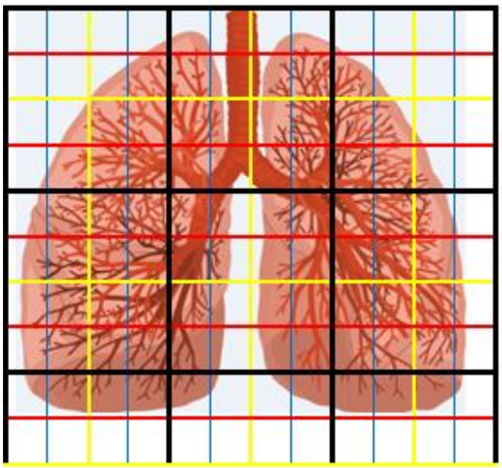

Figure 6. Three different sized cells are drawn over the human lung. From this it was determined that: a is the size of the cell, conditionally a = 48 mm, the number of cells in the drawing is corresponding, the number of cells in black is $N_1 = 6$, the number of cells in yellow is $N_2 = 26$, the number of cells in blue is $N_3 = 87$.

Based on the data in Figure 6, the human lung has a fractal structure and its fractal size is defined as follows:

Table 3. The results of the location of the vascular systems of the human lung in several cells

The size of the cell a	9	16	48
Number of cells N	89	27	8
$y = lnN$	4,4659	3,2580	2,7917
$x = lna$	2,1972	2,7726	3,8712

From Table 3, it can be seen from the values of N that the human lung has a complex structure. Because as the cell size decreases, the number of cells increases. This indicates an infinite distribution of blood vessels in the lungs. Based on the above data, when the fractal size of the human lung is calculated using the cell method.

According to formula 5 is equal to the following.

$$D = \left(\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i - n\sum_{i=1}^{n} x_i y_i\right) \Bigg/ \left(n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right) = $$

$$= 1,5626 \quad (8)$$

In addition, fractal measurements of the pulmonary vascular system of patients have been evaluated in several studies and are widely used to describe the vascular systems in various diseases. For example, computed tomography angiography can assess the deterioration in survival due to a decrease in the fractal size of the pulmonary arterial tree and an increased risk of insulin in people with pulmonary hypertension.

The vascular systems in the retina, which is another part of the human body with a fractal structure, provide oxygen to every tissue in the body and improve blood circulation, nourishment, and prevent tissue damage and functional impairment. Human blood vessels have the same fractal structure as above, and its fractal size can also be determined.

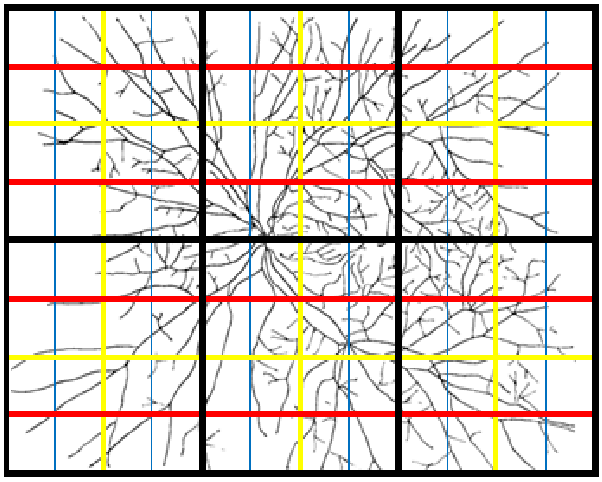

Figure 7. Three different sizes of cells are drawn over the vascular systems in the human retina. From this it was determined: a is the size of the cell, conditionally a = 48 mm, the number of cells in the drawing is corresponding, the number of cells in black is $N_1 = 6$, the number of cells in yellow is $N_2 = 24$, the number of cells in blue is $N_3 = 116$.

Based on the data in Figure 7, we determine the fractal size of the vascular systems in the human retina as follows:

Table 4. Results of the location of human vascular systems in multiple cells

The size of the cell a	9	16	48
Number of cells N	116	24	6
$y = lnN$	4,7536	3,1780	1,7917
$x = lna$	2,1972	2,7726	3,8712

The values of N in Table 5 show that the vascular systems in the human retina have a complex structure. Because the smaller the cell size, the higher the number of cells. This indicates that the vascular systems in the retina are infinitely distributed. Based on the above data, the fractal size of the vascular systems in the human retina is calculated using the grid method, i.e., according to formula 5, is as follows.

$$D = \left(\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i - n\sum_{i=1}^{n} x_i y_i\right) \Bigg/ \left(n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right) = $$

$$= 1,7021 \quad (9)$$

The vascular systems in the human retina have their own characteristics, and as a person grows older, the blood vessels in his retina grow like trees, like a king. This means that the blood vessels in the human retina change fractal size over time. However, the value of the fractal dimension does not change much relative to the original dimension. That is, the results of the study show that the fractal size of the blood vessels in the human retina varies by ± 0.073.

CONCLUSION

1. It was found that the human airways have a fractal structure, and on this basis the fractal size of the human lungs was found.

2. It was found that the fractal size of the human lung does not depend on body size, but varies from 1.57 to 1.58.

3. It was found that the vascular system in the human retina has a fractal structure.

4. The fractal size of the vascular systems in the human retina was found to be 1.7. Through this measurement, it is possible to predict whether Nisa has diabetes mellitus.

5. It was found that the fractal size of the vascular system in the human retina may change with age, i.e., its value may increase.

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