Available online at www.rajournals.in

RA JOURNAL OF APPLIED RESEARCH

ISSN: 2394-6709

DOI:10.31142/rajar/v4i10.11

Volume: 04 Issue: 10 October -2018

International Open Access

ICV- 74.25

Impact Factor :5.985

Page no.- 2071-2076

Study on Properties of IOWHA Operator Combination Forecasting Model Based on Exponential Support

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ARTICLE INFO

Published Online: 26 October 2018

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ABSTRACT

To overcome disadvantages of traditional single predication methods in selecting fixed parameters, exponential support was brought in based on IOWHA operator to construct optimal combined predication model of IOWHA operator based on exponential support and to find out the model of IOWHA operator based on mean dispersion that is in consistent with it for study. Moreover, clear definition were given to the predication accuracy as well as the superiority and non-inferiority of the model, and sufficient conditions of the existence of non-inferiority, superior combination forecasting of the model were explored from a theoretical perspective. The example analysis showed that this model was superior to traditional combination forecasting model, for it could use fully the information of each individual method and could improve the prediction accuracy of the model. In a word, it is a kind of superior combination forecasting.

KEYWORDS: Combination Forecasting; Exponential Support; IOWHA Operator

I. INTRODUCTION

Since individual prediction method is limited to extract useful information from one aspect, there are problems as low prediction accuracy. Therefore, C. W. J. Granger et al[1] put forward the idea of combined prediction, i.e., to combine properly various individual methods and to use reasonably and effectively the information revealed by them so as to achieve better prediction. Later, Yager R. R. and Filev D. put forward "orand" data information combined operator that exists between "or" and "and", that is, ordered weighted averaging (OWA) operator[2]. Because OWA operator and its expanding methods have the advantages to improve fitting precision and predictive ability of the model, it is applied extensively to artificial neural network, fuzzy system control and fuzzy modeling, information integration, decision analysis, combined prediction, communications network and etc. [3-9]. However, in most applications, error square and minimum are taken as the optimal criteria, so, Tang Xiaowo[10], Ma Yongkai[11], Chen Huayou[12] and Wang Yingming[13] put forward one after another the idea of superior combined forecasting, theorem of removing redundant, nature of superior model and the indicators of judging its superiority. Based on the above research, Chen Huayou[14] and Shuan Yan[15] combined vector angle consine and grey correlation degree with combination model and established respectively the optimal prediction model based on them; while Yager[16], considering the internal relation between the data in study, raised the concept of support. So, on the basis of combining exponential support and IOWHA support, this essay established the combined prediction model of IOWHA operator based on exponential support and found out the combination model based on average dispersion that was equivalent to the model. The existence of superior combination forecasting of the model was discussed to show the rationality and effectiveness of the combination method of IOWHA operator based on exponential support.

II. OPTIMAL COMBINATION FORECASTING MODEL OF IOWHA OPERATOR BASED ON EXPONENTIAL SUPPORT

A. *Symbol description and concepts*

Definition 1. Suppose $(<v_1,a_1>,<v_2,a_2>,\cdots,<v_n,a_n>)$ is a double dimensional array containing n data, IOWHA is *n*-variate functions, $W=(w_1,w_2,\cdots,w_n)^T$ is the weighted vectors corresponding to each element, and there is w in $\sum_{i=1}^{n} w_i = 1$ $w_i \geq 0$, $i=1,2,\cdots,n$.

Suppose

$$IOWHA(<v_1,a_1>,<v_2,a_2>,\cdots,<v_n,a_n>) = 1\Big/\sum_{i=1}^{n}\frac{w_i}{a_{v-index(i)}} \quad (1)$$

In Formula (1), $v_1,v_2,\cdots,v_n$ is the induced variable, $a_1,a_2,\cdots,a_n$ are elements of the original data set. First, it is necessary to rank $v_1,v_2,\cdots,v_n$ from big to small and suppose

$v-index(i)$ is the subscript of a that corresponds with the i large number. IOWHA is named n-dimensional induced ordered weighted and harmonic averaging operator and is shortened as IOWHA operator.

Definition 2. Suppose $a,b,x,y \in R^{+}$. If $Sup(a,b)$ satisfies the following conditions:

- $Sup(a,b) \in [0,1]$, if and only if $a=b$, $Sup(a,b)=1$
- $Sup(a,b) = Sup(b,a)$
- If $|a-b| < |x-y|$, $Sup(a,b) \ge Sup(x,y)$

Then, $Sup(a,b)$ is b's support to a.

Especially, a type of support is given in literature [16], that is,

$$Sup\left(\frac{a}{a+b}, \frac{b}{a+b}\right) = \exp\left(-\frac{|a-b|}{a+b}\right) \qquad (2)$$

It is named b's average index support to a and is recorded as $Sup(a,b)$.

If take in a definition 2 as an true value in certain economic phenomena and b as a prediction value of the economic phenomena, $Sup(a,b)$ could then be regarded as an indicator which could reflect the prediction accuracy. The more obvious the difference between data set and data set is, the small the average exponential support $Sup(a,b)$ between them is; moreover, if and only if data set b accord completely with data set a, the average exponential support $Sup(a,b)$ reaches the maximum value, i.e., $Sup(a,b)=1$.

Suppose the true value of certain phenomena is $\{x_t, t=1,2,\cdots,N\}$ and m types of individual prediction methods are used to predict it, x_{it} is used to signify the fitted value at time t with the i individual method, $i=1,2,\cdots,m$, $t=1,2,\cdots,N$. Since the data in source data set $\{x_t, t=1,2,\cdots,N\}$ and the data obtained with the prediction method are all dimensional data set, they require still the following unitary processing:

Definition 3. Suppose a_{it} is the prediction accuracy of i method at time t, among which:

$$a_{it} = \begin{cases} 1-|(x_t - x_{it})/x_t| & ; \quad |(x_t - x_{it})/x_t| < 1 \\ 0 & ; \quad |(x_t - x_{it})/x_t| \ge 1 \end{cases} \qquad (3)$$

Obviously, $a_{it} \in [0,1]$, $i=1,2,\cdots,m; t=1,2,\cdots,N$.

Take a_{it} as the induced value of y_{it} after x_{it} is processed unitarily, then m data groups are formed between a_{it} and y_{it} : $<a_{1t}, y_{1t}>, <a_{2t}, y_{2t}>, \cdots, <a_{mt}, y_{mt}>$, $i=1,2,\cdots,m$; $t=1,2,\cdots,N$. Order from big to small m types of individual prediction methods at time t, $a_{1t}, a_{2t}, \cdots, a_{mt}$. Suppose $a-index(it)$ is the subscript of y that corresponds to i after the ranking of $a_{1t}, a_{2t}, \cdots, a_{mt}$, $\hat{y}_t$ is the combined prediction value of IOWHA operator generated at time t with $a_{1t}, a_{2t}, \cdots, a_{mt}$ as the induced value, among which, $t=1,2,\cdots,N$. According to definition 1, the following formula could be obtained:

$$\hat{y}_t = IOWHA(<a_1, y_{1t}>, <a_2, y_{2t}>, \cdots, <a_m, y_{mt}>) = 1 \Big/ \sum_{i=1}^{m} \frac{w_i}{y_{a-index(it)}} \qquad (4)$$

Definition 4. Suppose $e_{it} = 1/y_t - 1/y_{it}$ is the predicted reciprocal error between the i individual method at time t and its corresponding true data set (after unitary processing); $e_{it} = 1/y_t - 1/\hat{y}_t$ stands for the predicted reciprocal error between the combined prediction data set at time t with its corresponding true data set (after unitary processing); $i=1,2,\cdots,m$; $t=1,2,\cdots,N$. If $e_{a-index(it)} = 1/y_t - 1/y_{a-index(it)}$, then

$$e_t = \frac{1}{y_t} - \frac{1}{\hat{y}_t} = \frac{1}{y_t} - \sum_{i=1}^{m} \frac{w_i}{y_{a-index(it)}} = \sum_{i=1}^{m} \frac{w_i}{y_t} - \sum_{i=1}^{m} \frac{w_i}{y_{a-index(it)}} = \sum_{i=1}^{m} w_i e_{a-index(it)} \qquad (5)$$

Apply exponential support to IOWHA operator, the following definition could then be obtained:

Definition 5. Take Sup_i as the support of the predicted value sequence of the i individual method and its true value sequence, and Sup is the support of IOWHA operator prediction value sequence and the true value sequence, among which:

$$Sup_i(y_t, y_{it}) = \exp\left(-\left(\sum_{t=1}^{N} |e_{it}|\right) \Big/ \left(\sum_{t=1}^{N} \frac{1}{y_t} + \sum_{t=1}^{N} \frac{1}{y_{it}}\right)\right) \qquad (6)$$

$$Sup(y_t, \hat{y}_t) = \exp\left(-\frac{\sum_{t=1}^{N} \left|\sum_{i=1}^{m} w_i e_{a-index(it)}\right|}{\sum_{t=1}^{N} \frac{1}{y_t} + \sum_{t=1}^{N} \sum_{i=1}^{m} \frac{w_i}{y_{a-index(it)}}}\right) \qquad (7)$$

It could be seen from formula (7), the support Sup of IOWHA prediction value sequence and the true value sequence is function of $W = (w_1, w_2, \cdots, w_m)^T$, recorded as $Sup(w_1, w_2, \cdots, w_m)$. So, the optimal combined prediction model of IOWHA operator based on exponential support is:

$$\max Sup(w_1, w_2, \cdots, w_m) = \exp\left(-\frac{\sum_{t=1}^{N} \left|\sum_{i=1}^{m} w_i e_{a-index(it)}\right|}{\sum_{t=1}^{N} \frac{1}{y_t} + \sum_{t=1}^{N} \sum_{i=1}^{m} \frac{w_i}{y_{a-index(it)}}}\right)$$

$$s.t. \begin{cases} w_1 + w_2 + \cdots + w_m = 1 \\ w_i \ge 0, i = 1,2,\cdots,m \end{cases}$$

Since exponential function is monotone funcion, the model mentioned above equals the following model:

$$\min f(w_1,w_2,\cdots,w_m)=\frac{\sum_{t=1}^{N}\left|\sum_{i=1}^{m}w_ie_{a-index(it)}\right|}{\sum_{t=1}^{N}\frac{1}{y_t}+\sum_{t=1}^{N}\sum_{i=1}^{m}\frac{w_i}{y_{a-index(it)}}} \quad (8)$$

$$s.t.\begin{cases}w_1+w_2+\cdots+w_m=1\\ w_i\ge 0, i=1,2,\cdots,m\end{cases}$$

Definition 6. Suppose

$$f_i=\left(\sum_{t=1}^{N}|e_{it}|\right)\Big/\left(\sum_{t=1}^{N}\frac{1}{y_t}+\sum_{t=1}^{N}\frac{1}{y_{it}}\right)$$

$$f(w_1,w_2,\cdots,w_m)=\frac{\sum_{t=1}^{N}\left|\sum_{i=1}^{m}w_ie_{a-index(it)}\right|}{\sum_{t=1}^{N}\frac{1}{y_t}+\sum_{t=1}^{N}\sum_{i=1}^{m}\frac{w_i}{y_{a-index(it)}}} \quad (9)$$

Take f_i as the average dispersion between the predicted value sequence of the i individual prediction method and its true value sequence; take $f(w_1,w_2,\cdots,w_m)$ as the average dispersion between the combined predicted value sequence of IOWHA operator and its true value sequence (after unitary processing). Mark $f_{\min}=\min\{f_i, i=1,2,\cdots,m\}$, $f_{\max}=\max\{f_i, i=1,2,\cdots,m\}$; If $f(w_1,w_2,\cdots,w_m)< f_{\min}$, then the combined prediction model determined by the weighted coefficient $w_1,w_2,\cdots,w_m$ is superior combined forecasting; if $f_{\min}\le f(w_1,w_2,\cdots,w_m)\le f_{\max}$, it is called non-inferiority combined forecasting; if $f(w_1,w_2,\cdots,w_m)> f_{\max}$, it is called inferiority combined forecasting.

III. NON-INFERIORITY AND OPTIMALITY OF COMBINED FORECASTING MODEL

Theorem 1. In the combination forecasting methods (8), equally weighted averaging combination forecasting methods at least are non-inferior combination forecasting methods.

Proof. Assume $W=(w_1,w_2,\cdots,w_n)^T$ where is any feasible solution of combination forecasting methods (8), where $w_1+w_2+\cdots+w_m=1$, $w_1,w_2,\cdots,w_m\ge 0$.

Then average exponential supporting degree is

$$f(w_1,w_2,\cdots,w_m)=\frac{\sum_{t=1}^{N}\left|\sum_{i=1}^{m}w_ie_{a-index(it)}\right|}{\sum_{t=1}^{N}\frac{1}{y_t}+\sum_{t=1}^{N}\sum_{i=1}^{m}\frac{w_i}{y_{a-index(it)}}}$$

We have

$$\sum_{t=1}^{N}\left|\sum_{i=1}^{m}w_ie_{a-index(it)}\right|$$

$$=f(w_1,w_2,\cdots,w_m)\cdot\left(\sum_{t=1}^{N}\frac{1}{y_t}+\sum_{t=1}^{N}\sum_{i=1}^{m}\frac{w_i}{y_{a-index(it)}}\right)$$

By Jensen's inequality, we have

$$\sum_{t=1}^{N}\left|\sum_{i=1}^{m}w_ie_{a-index(it)}\right|\le\sum_{t=1}^{N}\sum_{i=1}^{m}w_i\left|e_{a-index(it)}\right|=\sum_{i=1}^{m}w_i\sum_{t=1}^{N}|e_{it}|$$

By (9), we have

$$f(w_1,w_2,\cdots,w_m)\cdot\left(\sum_{t=1}^{N}\frac{1}{y_t}+\sum_{t=1}^{N}\sum_{i=1}^{m}\frac{w_i}{y_{a-index(it)}}\right)$$

$$\le\sum_{i=1}^{m}w_if_i\cdot\left(\sum_{t=1}^{N}\frac{1}{y_t}+\sum_{t=1}^{N}\frac{1}{y_{it}}\right)$$

When the combination forecasting method has equal weights, that is

$$w_1=w_2=\cdots=w_m=1/m$$

$$f\left(\frac{1}{m},\frac{1}{m},\cdots,\frac{1}{m}\right)\cdot\left(\sum_{t=1}^{N}\frac{1}{y_t}+\sum_{t=1}^{N}\sum_{i=1}^{m}\frac{1}{y_{a-index(it)}}\right)$$

$$\le\sum_{i=1}^{m}\frac{1}{m}f_i\cdot\left(\sum_{t=1}^{N}\frac{1}{y_t}+\sum_{t=1}^{N}\frac{1}{y_{it}}\right)$$

Then

$$f\left(\frac{1}{m},\frac{1}{m},\cdots,\frac{1}{m}\right)\le f_{\max}\cdot\frac{\sum_{t=1}^{N}\frac{1}{y_t}+\sum_{t=1}^{N}\frac{1}{y_{it}}}{\sum_{t=1}^{N}\frac{1}{y_t}+\sum_{t=1}^{N}\sum_{i=1}^{m}\frac{1}{y_{a-index(it)}}}$$

$$=f_{\max}\cdot 1=f_{\max}$$

So form Definition 6 we know that equally weighted averaging combination forecasting methods at least are non-inferior. This theorem is proved.

Theorem 2. In the combination forecasting methods (8), if $|e_{it}|<|e_{1t}|$, $y_{it}<y_{1t}$, $i=2,3,\cdots,m)$, $t=1,2,\cdots,N$, then the combination forecasting corresponding to any feasible solution of model (8) at least is non-inferior combination forecasting.

Proof. Assume $W=(w_1,w_2,\cdots,w_n)^T$ ($w_1+w_2+\cdots+w_m=1$, $w_1,w_2,\cdots,w_m\ge 0$) is any feasible solution of model (8), then

$$\sum_{t=1}^{N}\left|\sum_{i=1}^{m}w_ie_{a-index(it)}\right|\le\sum_{t=1}^{N}\sum_{i=1}^{m}w_i\left|e_{a-index(it)}\right|=\sum_{t=1}^{N}|e_{1t}|$$

Since

$$|e_{it}|<|e_{1t}|, y_{it}<y_{1t}; (i=2,3,\cdots,m; t=1,2,\cdots,N)$$

That is

$$\left|e_{a-index(it)}\right|<|e_{1t}|, y_{a-index(it)}<y_{1t}$$

$$f_i<f_1=f_{\max}=\max\{f_1, f_2,\cdots, f_m\}$$

Then we have

$$f(w_1, w_2, \cdots, w_m) = \frac{\sum_{t=1}^{N} \left| \sum_{i=1}^{m} w_i e_{a-index(it)} \right|}{\sum_{t=1}^{N} \frac{1}{y_t} + \sum_{t=1}^{N} \sum_{i=1}^{m} \frac{w_i}{y_{a-index(it)}}}$$

$$\leq \frac{\sum_{t=1}^{N} |e_{1t}|}{\sum_{t=1}^{N} \frac{1}{y_t} + \sum_{t=1}^{N} \frac{1}{y_{1t}}} = f_1 = f_{\max}$$

The conclusion can be achieved by Definition 6, which ends the proof.

Theorem 3. In the combination forecasting methods (8), if

$$\sum_{t=1}^{N} y_{a-index(it)} < \sum_{t=1}^{N} y_{it}$$

then the model corresponding to the optimal solution of model (8) must be the superior combination forecasting.

Proof. Assume

$$f_{\min} = f_k = \left(\sum_{t=1}^{N} |e_{kt}| \right) \Big/ \left(\sum_{t=1}^{N} \frac{1}{y_t} + \sum_{t=1}^{N} \frac{1}{y_{kt}} \right)$$

Time t is the forecasting accuracy of $y_{v-index(1t)}$ is the largest, then

$$\left| x_t - x_{a-index(1t)} \right| \leq |x_t - x_{it}|, \left| e_{a-index(1t)} \right| \leq |e_{it}| 。$$

That is

$$\sum_{t=1}^{N} y_{a-index(it)} < \sum_{t=1}^{N} y_{it}, \sum_{t=1}^{N} y_{a-index(1t)} < \sum_{t=1}^{N} y_{kt}; (k \neq 1)$$

Since $W^* = \left(w_1^*, w_2^*, \cdots, w_m^*\right)^T$ is the optimal solution of the combined prediction model (8), and $W = (1,0,0,\cdots,0)^T$ is the feasible solution,then

$$f\left(w_1^*, w_2^*, \cdots, w_m^*\right) \leq f(1,0,0,\cdots,0)$$

$$= \frac{\sum_{t=1}^{N} \left| e_{a-index(1t)} \right|}{\sum_{t=1}^{N} \frac{1}{y_t} + \sum_{t=1}^{N} \frac{1}{y_{a-index(1t)}}} \leq \frac{\sum_{t=1}^{N} |e_{kt}|}{\sum_{t=1}^{N} \frac{1}{y_t} + \sum_{t=1}^{N} \frac{1}{y_{kt}}} = f_k = f_{\min}$$

The conclusion can be achieved by Definition 6, which ends the proof.

IV. JOINT FORECASTING OF AVERAGE WAGE OF EMPLOYEES

To explain the rationality and effectiveness of IOWHA operator model based on exponential support, the true value of the average wages of staff in some province between 1999-2008 was adopted for example analysis by constructing combined prediction model with three individual predictions respectively, i.e., grey prediction model, exponential smoothing model and quadratic curve function model (see Table 1). Formula (3) is adopted to calculate the accuracies of each individual method and the data obtained are processed unitarily (see Table 1). Substitute the corresponding data ranked from big to small according to their accuracies into formula (4) and obtain the expected value of IOWHA operator of each timing. Substitute the expected value obtained into formula (8) and obtain the combined prediction model of IOWHA operator based on average dispersion. The optimum weighting coefficients could be obtained with LINGO software, which are respectively $w_1 = 0.625242$, $w_2 = 0.373087$, $w_3 = 0.001671$. Substitute weighting coefficient and the data in Table 2 into formula (1) and the combined prediction value could be obtained (see Table 3).

Table 1. The actual average wages of the staff and the predicted values obtained with three individual methods (unit: yuan)

Year	Actual value	Grey prediction model	Exponential smoothing model	quadratic curve function model
1999	6516.0	6516.0	6921.2	6081.0
2000	6989.0	6825.9	7292.5	7323.1
2001	7908.0	7595.2	8094.7	8749.0
2002	9296.0	8940.7	9389.0	10358.8
2003	10581.0	10840.2	10352.3	12152.3
2004	12928.0	13608.7	11035.7	14129.7
2005	15334.0	15637.9	13119.6	16290.9
2006	17949.0	18248.5	17821.0	18635.9
2007	22180.0	21671.4	21405.1	21164.7
2008	26363.0	25380.2	25902.8	23877.3

Table 2. Actual value of the index and the predicted value obtained after ranking from big to small according to the accuracy and unitary processing as well as its prediction accuracy

y_t	$y_{a-index(1t)}$	Accur-acy	$y_{a-index(2t)}$	Accur-acy	$y_{a-index(3t)}$	Accur-acy
0.941	0.942	1.000	1.000	0.938	0.879	0.933
0.954	0.932	0.977	0.996	0.957	1.000	0.952
0.904	0.925	0.976	0.868	0.960	1.000	0.894
0.897	0.906	0.990	0.863	0.962	1.000	0.886
0.871	0.852	0.978	0.892	0.976	1.000	0.852
0.915	0.963	0.947	1.000	0.907	0.781	0.854
0.941	0.960	0.980	1.000	0.938	0.805	0.856
0.963	0.979	0.993	1.000	0.983	0.956	0.962
1.000	0.977	0.977	0.965	0.965	0.954	0.954
1.000	0.983	0.983	0.963	0.963	0.906	0.906

Table 3. Combined prediction value of IOWHA operator based on average dispersion

Year	1999	2000	2001	2002	2003
Actual value	6081	7323.1	8749	10358.8	12152.3
IOWHA prediction value	6516	6989	8087.5	9437.8	10699
Year	2004	2005	2006	2007	2008
Actual value	11035.7	13119.6	17821	21164.7	23877.3
IOWHA prediction value	13212	15289	18253	21537	25455

According to the evaluation principle of combined prediction, select five errors that could evaluate the accuracy of the model for comparison, the result of which see Table 4.

- Error of sum square: $SSE = \sum_{i=1}^{N}(x_t - \hat{x}_t)^2$
- Mean square error: $MSE = \frac{1}{N}\sqrt{\sum_{i=1}^{N}(x_t - \hat{x}_t)^2}$
- Mean absolute error: $MAE = \frac{1}{N}\sum_{i=1}^{N}\left|x_t - \hat{x}_t\right|$
- Mean absolute percentage error: $MAPE = \frac{1}{N}\sum_{t=1}^{N}\left|\frac{(x_t - \hat{x}_t)}{x_t}\right|$
- Mean square percent error: $MSPE = \frac{1}{N}\sqrt{\sum_{t=1}^{N}\left(\frac{x_t - \hat{x}_t}{x_t}\right)^2}$

Table 4. Effectiveness evaluation of individual prediction and combined prediction method

Error type	Prediction method 1	Prediction method 2	Prediction method 3	Error square and minimum combined prediction	IOWHA combined prediction
SSE	1238300	4945500	20317000	1506806	1179300
MSE	292.44	623.92	1198	264.1	102.33
MAE	111.28	222.38	450.74	122.7	101.62
MAPE	0.020811	0.047373	0.092172	0.0157	0.015438
MSPE	0.000595	0.002599	0.010649	0.006	0.0003677

It could be seen from Table 4, compared with three individual methods and the error square and minimum combination methods, the value obtained from IOWHA combination method based on average dispersion is the smallest in the five errors mentioned above, which shows that IOWHA combined prediction method based on average dispersion is better than three individual prediction methods and the error square and the minimum combined prediction method and could obtain results with higher accuracy.

Besides, with formula (9), the average dispersion between the prediction value and the true value of three individual prediction methods and the combined prediction methods could be obtained, which are respectively: $f_1 = 0.013871$, $f_2 = 0.027512$, $f_3 = 0.040005$, $f = 0.012657$. It could be seen thus the average dispersion value obtained with the combined prediction method is smaller than that obtained with individual prediction method, that is, both satisfy $f < \min(f_1, f_2, f_3)$ in definition 6, which suggests the IOWHA combined prediction model based on average dispersion is superior combination forecasting.

V. Conclusion

Based on definition of support, this essay combines exponential support and IOWHA operator and constructs IOWHA operator combined prediction model based on average dispersion which equals the optimal prediction model based on exponential support. Sufficient conditions of the existence of the superior combination forecasting are studied theoretically. Example analysis shows that this model is superior to traditional combination prediction model, for it could take full use of each individual prediction method and improve the prediction accuracy. In a word, it is a superior combination forecasting.

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