



Analysis of Supply Chain Optimization Model Based on new Cooperative Order Agriculture Model

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ARTICLE INFO

Published Online:
12 June 2018

ABSTRACT

This paper builds a new order agriculture supply chain model based on the characteristics of a new mode of order agriculture. Firstly, establish an agriculture cooperative in accordance with the development of regional economy and farmers buy a share according to the land area. Then the agriculture cooperative gets profits by signing a contract with agricultural products acquiring company. Part of the income is used to pay dividends for farmers. Besides, to ensure the interests of the farmers, the cooperative will give the farmers a fixed income according to the stock. Based on these, we build a new three-level order agricultural supply chain consisting of a company, a cooperative and farmers. And calculate the income of cooperative under different risk aversions using the measure of conditional value-at-risk (CVaR). Then set up a constraint optimization model, obtain the minimum land area that farmers participate in cooperative can get more interests and the minimum fixed income that cooperative provides by the Lagrange function and relevant KKT conditions.

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KEYWORDS: new mode, cooperative mode, supply chain model, CVaR Lagrange function

1. Introduction

With the gradual deepening of China's rural economic system reform, the mode of agricultural production and operation is also changing to meet the needs of the new market economy development. A new type of production and management mode, "farmers+cooperative+company", which is a new three-level order agricultural supply chain has grown up.

Order agriculture, which is also called contract farming, connects the agricultural production and the market through the contract. That is farmers sign the contract which including the varieties, quality, price, quantity, delivery date of agricultural productions and responsibility and obligation of participants with agricultural leading enterprise before production. Then farmers grow produce according to the contracts^[1].

At present, there are five main forms of order agriculture in our country:

(1) the farmers+leading enterprises, mainly relying on the

leading enterprises or introducing foreign capital, are led by the company and signed the contract of production and marketing with farmers.

- (2) the farmers+intermediary or brokers, relying on intermediary organizations to develop agricultural production and management.
- (3) the farmers+professional wholesale market, mainly relying on the wholesale market to develop the production and sales of agricultural products.
- (4) the farmers+scientific research market, mainly relying on the scientific research and technology service departments to sign the contract of crop seed production, and develop the production and marketing of seed industry.
- (5) Agricultural technology departments or merchants, agricultural technology departments organize production order agricultural products.

Among these, the form of "farmers+leading enterprises"

started in nineteenth century from 30s to 40s in foreign countries, which has a long history. It is widely adopted by developed countries as a kind of management mode that can effectively link the purchase and sale chain of agricultural products. In the past 1930-1950 years, the order agriculture developed rapidly in Europe and America. The United States, The United Kingdom, Canada, France, Holland, Hungary and other countries have participated in it. The agricultural products involved in the order are mainly concentrated on grain, animal husbandry, fruit, canned vegetable and seed production. So order agriculture has become one of the basic features of modern agriculture in developed countries at the end of the twentieth century.

Compared with the developed countries, China's order agriculture started late and originated in the southeast coastal area of 1980s. Order agriculture has effectively linked production and marketing, which has changed the phenomenon of farmers' blind follow up in the past, and solved the problem of sales of products, so it has been welcomed by farmers. But compared with the complete order agricultural system in foreign countries, the present order agriculture in China is still in the stage of trial and exploration. Because there is no complete financial environment support, no special laws and the regulatory system is still in a blank state. Which leads to the increasingly serious problems exposed in the development of China's order agriculture, the most prominent problem is the low order performance rate. The survey by Jinyong Guo^[2] in Jiangxi Province showed that only 38.3% of the farmers fulfilled the contract and included farmers without full compliance. Aiqun Wang and Ying Xia^[3] studies show that the proportion of contract modes is the largest among the various modes of agricultural industrialization in China. Among them, the contract between farmers and the national leading enterprises is 86%, but the order performance rate is only 20%. The main reasons are the following two aspects:

Firstly, it's inseparable from the basic situation of farmers in China. As we all know, there are a large number of farmers, but the production scale of individual farmers is small, which results in the weak position of farmers in the order agriculture. The weak negotiation ability, the risk aversion of farmers themselves and the factors such as price and output of agricultural products will all affect farmers' decision-making.

Secondly, it's also related to the unique characteristics of order agriculture. The seasonal characteristics of agricultural products are very strong. They have a long product lead time

but a short inventory time. The sales season is short and the salvage value is low. Furthermore, there are many factors that influence the output such as season, weather, natural disasters, pests and disease and so on. Besides, prices are also affected by the market volatility. All of these features will affect the setting of order agricultural contracts^[4].

In the empirical research of Jiujiu Ma, Xuegao Xu^[5], shows that only under the condition of high equilibrium strength, order agriculture can achieve low cost performance rate. But the small scale of production and risk aversion of farmers make farmers and companies obviously unbalanced. The research of Bin Li and Mingzhi Liu^[6] also confirms this. The biggest impact of the company+farmers mode on the whole contract agriculture is the risk from farmers, while the risk from the external market is relatively mild.

Therefore, the low order performance rate has become the core problem of order agriculture to be solved urgently, so many scholars have studied this problem. The research from the economics analysis by Yan Hu^[7] shows that the existence of moral hazard and opportunism has led to the low performance of the order due to the asymmetric information. Zylbersztajn^[8] shows that the performance rate of farmers is in direct proportion to the scale of their production from the analysis of the influence factors. Sartwelle et al.^[9] studies shows that farmers who are close to the market are more likely to default. The empirical research by Hongdong Guo^[10] shows that the order fulfillment rate of implementing the strategy of “prices change with the market” is much higher than exercising the other price strategies. And it's also believed that the rate of order performance for farmers with special requirements and incentive measures is higher than the rate of order performance without these provisions.

The reason of the low order performance comes down to the facts that the unfair distribution of profit and the unbalance of risk-taking between farmers and company. Order agriculture not only has the risk of agriculture itself, such as natural disasters, price fluctuations and other natural risks and market risks, but also has institutional risk and credit risk. Liangyuan Sun, Yueheng Zhang^[11] through the analysis of the characteristics of China's agricultural transformation from planned economy to market economy and gradually into the perfect market economy and pointed out that the natural risk of agriculture has given place to the market risk and institutional risk. Fengqin Liu^[12], through the study of large order agricultural cases, further pointed out that the moral risk of the

order agricultural contract is the hidden action. Leading enterprise entices farmers to sign the unreasonable contract by hiding knowledge, action, which pushes the market risk to farmers and then leads to the credit risk. Jing Liu^[13] pointed out that the price agreement reached by both parties does not let the market risk out, but transfers between each other. Considering the China farmers are scattered individual farmers, they are unable to attend the rapid change of price and variety of agricultural products, as well as the scarce information of cultivation, sales channels and price fluctuation of agricultural products. A lot of information is not available to farmers or is very expensive to get. Therefore, the great sales risk and price risk are difficult to bear for farmers who have only a few acres of land. On the other hand, acquiring company, as a leader, has its absolute advantage in the transaction with farmers, because of its decision-making system, organization system, perfect marketing system and communication ability with the government. As a result, scattered farmers are very hard to freely choose their trading partners to sign contracts voluntarily, and often have to accept supplementary contracts provided by agricultural related enterprises with market monopoly. (Zuhui Huang^[14], Xiaxian Fu^[15])

To sum up, the interests of farmers are not guaranteed, the risk is too high, and the risk aversion of farmers themselves, all of which lead to a serious breach of contract farming. On the surface, the breach of contract is the short-term behavior between farmers and enterprise, but its intrinsic root is that the mechanism of interest risk is not sound. (Zhaosheng Xue^[16]) Hence, it is necessary to establish a coordinated contract mechanism between farmers and company.

The investigation and study found that many places adopted the agricultural model of “farmers+cooperative+company”. Yanping Zhang^[17]'s research shows that small-scale farmers can increase the scale of production by joining cooperative,

which improves farmers' signing rate and ensures the performance rate. It can be seen that the establishment of cooperatives can not only improve farmers' order fulfillment rate, but also ensure farmers' income and reduce the risk of farmers, which is beneficial for farmers and acquiring company. For cooperatives, signing contracts with acquiring company has a bigger voice than a single farmer, and the risk is much smaller than that of farmers because of a variety of agricultural products. Further, nowadays the Internet is so developed that cooperatives can set up a brand, use the Internet to publicize and create brand effect, which can not only attract the attention of people, but also attract more acquiring company and strive for more resources for cooperatives. The rational distribution of the interests of the cooperatives and the guarantee of the income of farmers are the key to attract farmers to join the cooperative. The uncertainty of the agricultural product market makes the production plan of agricultural products also the key to determine the profitability of the cooperatives.

Based on the above situation, this paper puts forward a new type agricultural model of “farmers+cooperative+company”. That is, farmers can join the cooperative and share the income according to the proportion of their land area. Moreover, the cooperative will provide a fixed interests to farmers according to the size of land area, so as to ensure the basic interests of farmers. The cooperative will hire employees to grow crops, and farmers can also choose to be employees so that they can get an extra salary. Build a constraint optimization model on the basis of the supply chain model. Then calculate the minimum land area for farmers to choose to join the cooperative can gain more benefits and the minimum fixed interests provided by cooperative to ensure the income of farmers by Lagrange function, which provides a theoretical basis for the establishment of cooperatives.

2. Model

First, the relevant assumptions are put forward:

- (1) The cooperative will grow N ($N > 1$) kinds of crops and each yield is Q_i ($i = 1 \dots N$).
- (2) The market demand for agricultural products is x_i . (x_i is a non-negative random variable and its distribution function is

$F(\cdot)$, probability density function is $f(\cdot)$)

- (3) The prices of agricultural products are as follows:

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- a. Order price: w_i^r ;
 - b. Market purchasing price: w_i (w_i is a non-negative random variable and its distribution function is $G(\cdot)$, probability density function is $g(\cdot)$. $w_i \in [A_i, B_i]$)
 - c. Selling price: p_i . (p_i is a non-negative random variable and its distribution function is $H(\cdot)$, probability density function is $h(\cdot)$, $p_i \in [C_i, D_i]$);
- (4) The cost of growing agricultural products by cooperative is $C(Q_i)$ ($i = 1 \dots N$);
- (5) The cooperative will employ workers to grow agricultural products and the wages of workers are proportional to the production of agricultural products in order to increase the enthusiasm of workers. That is the whole wages of workers are $\sum_{i=1}^N c_{i3}Q_i$ ($c_{i3} > 0$). Farmers can apply to become workers. If the total number of workers is a and among them the number of farmers is b ($0 \leq b \leq a$)
- (6) The fixed interests that cooperative provides for farmers is M yuan per unit area.
- (7) The total area of all farmers joining the cooperative is S ;
- (8) Agricultural products are not easy to be preserved and its selling season is short, so we ignore the final value of the products in this paper.
- (9) For each kind of agricultural products, the company will purchase from cooperative according to the rule of “price changing with the market”. That is, at the end of production season, if the order price is less than the market purchasing price, then company will purchase the products at the market purchasing price; otherwise, the company will purchase at the order price.

Based on the above hypothesis, we give the revenue functions as follows^[18]:

The cooperative:

$$\pi_e = \varphi \sum_{i=1}^N \left[Q_i \max(w_i, w_i^r) - C(Q_i) - c_{i3}Q_i \right] - MS \tag{1}$$

Farmers:

$$\pi_f = (1 - \varphi) \sum_{i=1}^N \left[Q_i \max(w_i, w_i^r) - C(Q_i) - c_{i3}Q_i \right] + MS + \frac{b \sum_{i=1}^N c_{i3}Q_i}{a} \tag{2}$$

Company:

$$\pi_m = \sum_{i=1}^N \left[p_i \min(Q_i, x_i) - Q_i \max(w_i, w_i^r) \right] \tag{3}$$

The farmers here refer to all the farmers who joined the cooperative. For a single farmer, if the land area is S_1 ($S_1 = S/n$) ($n > 1$)

and the farmer chose to be a worker, then the farmer’s revenue function follows:

$$\pi_{f1} = \frac{1}{n} (1 - \varphi) \sum_{i=1}^N \left[Q_i \max(w_i, w_i^r) - C(Q_i) - c_{i3}Q_i \right] + MS_1 + \frac{\sum_{i=1}^N c_{i3}Q_i}{a} \tag{4}$$

3. Decision Models

Although the cooperative has a certain anti-risk ability compared to individual farmers, it is not risk preference. Therefore, the cooperative should not only consider the expected income of agricultural products, but also take into account the risk of income in the production plan. Therefore, we use the conditional value-at-risk (CVaR) to measure the average income of cooperative below the quantile η .^[19]Then considering the decision behavior of farmers and cooperative in this case.

A specific definition of CVaR is given below:

$$CVaR_{\eta} (g(x, y)) = E \left[g(x, y) \mid g(x, y) \leq q_{\eta}(y) \right] = \frac{1}{\eta} \int_{g(x,y) \leq q_{\eta}} g(x, y) p(y) dy \quad (5)$$

Here, E is the expectation function; $\eta \in (0,1]$ is the degree of risk aversion of decision-makers; $g(x, y)$ is the revenue function;

$p(y)$ is the probability density function of the random variable y ; q_{η} is the quantile of y . That is

$$q_{\eta}(y) = \sup \{v \mid \Pr \{g(x, y) \leq v\} \leq \eta\} \quad (6)$$

A more general definition of CVaR is given as below^[20]:

$$CVaR_{\eta} (g(x, y)) = \max_{v \in R} \left\{ v + \frac{1}{\eta} E \left[\min (g(x, y) - v, 0) \right] \right\} \quad (7)$$

Then we calculate according to the definition.

The revenue function of cooperative is given by equation (1). The φ, M and S in (1) are the fixed value defined by farmers or cooperative. Therefore, the revenue function of a kind of products follows:

$$\pi_i = Q_i \max (w_i, w_i^r) - C(Q_i) - c_{i3}Q_i \quad (8)$$

Then we calculate the CVaR of π_i . Let $g(Q_i, v) = v + \frac{1}{\eta} E \left[\min (\pi_i - v, 0) \right]$, that is

$$g(Q_i, v) = v - \frac{1}{\eta} \int_{A_i}^{B_i} \left[v - (Q_i \max (w_i^r, w_i) - C(Q_i) - c_{i3}Q_i) \right]^+ dG(w_i) \quad (9)$$

For ease of calculation, let $D(Q_i) = C(Q_i) + c_{i3}Q_i$, then

$$g(Q_i, v) = v - \frac{1}{\eta} \int_{A_i}^{w_i^r} \left[v - (Q_i w_i^r - D(Q_i)) \right]^+ dG(w_i) - \frac{1}{\eta} \int_{w_i^r}^{B_i} \left[v - (Q_i w_i - D(Q_i)) \right]^+ dG(w_i) \quad (10)$$

The maximum value of $g(Q_i, v)$ is calculated in three cases:

(a) $v < Q_i w_i^r - D(Q_i)$, it follows $g(Q_i, v) = v$ and $\frac{\partial g(Q_i, v)}{\partial v} = 1 > 0$

(b) $Q_i w_i^r - D(Q_i) \leq v < B_i Q_i - D(Q_i)$, it follows

$$g(Q_i, v) = v - \frac{1}{\eta} \int_{A_i}^{w_i^r} \left(v - (Q_i w_i^r - D(Q_i)) \right) dG(w_i) - \frac{1}{\eta} \int_{w_i^r}^{Q_i} \frac{v + D(Q_i)}{Q_i} \left(v - (Q_i w_i - D(Q_i)) \right) dG(w_i) \quad (11)$$

$$\frac{\partial g(Q_i, v)}{\partial v} = 1 - \frac{1}{\eta} G(w_i^r) - \frac{1}{\eta} \left(G\left(\frac{v + D(Q_i)}{Q_i}\right) - G(w_i^r) \right) = 1 - \frac{1}{\eta} G\left(\frac{v + D(Q_i)}{Q_i}\right) \quad (12)$$

$$\frac{\partial g(Q_i, v)}{\partial v} \Big|_{v=Q_i w_i^r - D(Q_i)} = 1 - \frac{1}{\eta} G(w_i^r) \quad (13)$$

$$\frac{\partial g(Q_i, v)}{\partial v} \Big|_{v=B_i Q_i - D(Q_i)} = 1 - \frac{1}{\eta} G(B_i) < 0 \quad (14)$$

(c) $v > B_i Q_i - D(Q_i)$, then

$$g(Q_i, v) = v - \frac{1}{\eta} \int_{A_i}^{w_i^r} \left(v - (Q_i w_i^r - D(Q_i)) \right) dG(w_i) - \frac{1}{\eta} \int_{w_i^r}^{B_i} \left(v - (Q_i w_i - D(Q_i)) \right) dG(w_i) \quad (15)$$

$$\frac{\partial g(Q_i, v)}{\partial v} = 1 - \frac{1}{\eta} G(w_i^r) - \frac{1}{\eta} \left(G(B_i) - G(w_i^r) \right) = 1 - \frac{1}{\eta} G(B_i) < 0 \quad (16)$$

Assume v^* is the optimal value of the maximum of $g(Q_i, v)$. It is known from the above three cases that $g(Q_i, v)$ is a concave function of v . Therefore,

if $1 - \frac{1}{\eta} G(w_i^r) \leq 0$ that is $0 < G^{-1}(\eta) \leq w_i^r$, we can obtain $v^* = w_i^r Q_i - D(Q_i)$ and

$$g(Q_i, v) = v^* = w_i^r Q_i - D(Q_i) \quad (17)$$

if $1 - \frac{1}{\eta} G(w_i^r) > 0$ that is $w_i^r < G^{-1}(\eta) \leq B_i$ there is a v^* that satisfies

$$1 - \frac{1}{\eta} G\left(\frac{v^* + D(Q_i)}{Q_i}\right) = 0$$

To solve it, $v^* = Q_i G^{-1}(\eta) - D(Q_i)$ and

$$g(Q_i, v) = w_i^r Q_i - \frac{1}{\eta} \int_{w_i^r}^{G^{-1}(\eta)} (w_i - w_i^r) Q_i dG(w_i) - D(Q_i) \quad (18)$$

In summary,

$$g(Q_i, v) = \begin{cases} w_i^r Q_i - D(Q_i), & 0 < G^{-1}(\eta) \leq w_i^r \\ w_i^r Q_i - \frac{1}{\eta} \int_{w_i^r}^{G^{-1}(\eta)} (w_i - w_i^r) Q_i dG(w_i) - D(Q_i), & w_i^r < G^{-1}(\eta) \leq B_i \end{cases} \quad (19)$$

$$\pi_e = \begin{cases} \varphi \sum_{i=1}^N \left(w_i^r Q_i - D(Q_i) \right) - MS, & 0 < G^{-1}(\eta) \leq w_i^r \\ \varphi \sum_{i=1}^N \left(w_i^r Q_i - \frac{1}{\eta} \int_{w_i^r}^{G^{-1}(\eta)} (w_i - w_i^r) Q_i dG(w_i) - D(Q_i) \right) - MS, & w_i^r < G^{-1}(\eta) \leq B_i \end{cases} \quad (20)$$

Next, we discuss the decision-making behavior of farmer and cooperative under the two cases.

3.1 $0 < G^{-1}(\eta) \leq w_i^r$

3.1.1 The Decision-Making Behavior of Farmer

The study by Fei Ye, Qiang Lin^[21] shows that the revenue function of individual farmer without cooperative is

$$\pi_{f2} = Q \max(w, w^r) - C(Q) \quad (21)$$

Here, Q is the yield of the farmer and assume the per unit yield is q ; w is the market purchasing price of the produce; w^r is the order price between the farmer and company. $C(Q) = c_0 + c_1Q + c_2Q^2$ is the cost function of Q and c_0 stands for the fixed cost of farmer such as the tool wear; c_1 is the input ratio of planting agricultural products, such as seeds, fertilizers, pesticides, etc; c_2 is the effort ratio of farmers.

So for a farmer with the land area S_1 , should he choose to participate in the cooperative or contract with company in an individual form? If the farmer does not have labor, of course he will choose to join the cooperative. So it is assumed that the farmer has a labor. The particular calculation is made below.

Constraint model^[22]:

$$\min z = MS_1 \quad (22)$$

$$s.t. -\pi_e < 0, \quad (23)$$

$$\pi_{f2} - \pi_{f1} < 0. \quad (24)$$

That is

$$\min z = MS$$

$$s.t. MS_1 - \varphi \sum_{i=1}^N \left(Q_i w_i^r - D(Q_i) \right) < 0 \quad (25)$$

$$S_1 q \max(w^r, w) - C(S_1 q) - \frac{1}{n} (1 - \varphi) \sum_{i=1}^N \left[Q_i \max(w_i^r, w_i) - D(Q_i) \right] - MS_1 - \frac{\sum_{i=1}^N c_{i3} Q_i}{a} < 0 \quad (26)$$

Here $D(Q_i) = C(Q_i) + c_{i3}Q_i$ and $C(Q_i)$ is the cooperative's cost function of planting. Assume that

$C(Q_i) = c_{i0} + c_{i1}Q_i + c_{i2}Q_i^2$. c_{i0} is the fixed cost such as the loss of machines; c_{i1} is the input ratio of growing crops in

cooperative, such as seeds, fertilizer, pesticide and so on. c_{i2} is the effort ratio of cooperative.

Let

$$L(S_1, \lambda, \mu) = z + \lambda(-\pi_e) + \mu(\pi_{f2} - \pi_{f1}) \quad (27)$$

Which is the Lagrange function of constraint model (22)(23)(24).And the relevant KKT conditions are

$$\frac{\partial L(S_1, \lambda, \mu)}{\partial S_1} = M + \lambda(Mn) + \mu(q \max(w^r, w) - c_1q - 2c_2q^2 S_1 - M) = 0 \quad (28)$$

$$\frac{\partial L(S_1, \lambda, \mu)}{\partial M} = S_1 + \lambda(nS_1) + \mu(-S_1) = 0 \quad (29)$$

$$\lambda(-\pi_e) = 0, \lambda \geq 0 \quad (30)$$

Obviously, the revenue function of cooperative must be positive, that is $-\pi_e < 0$. Hence, we can get $\lambda = 0$ from the equation (30).

Thus, solve the equations (28)(29) and we can obtain

$$\mu = 1$$

$$S_1 = \frac{\max(w^r, w) - c_1}{2c_2q} \quad (31)$$

Corollary 1. If the M, ϕ and $Q_i (i = 1 \dots N)$ which set by the cooperative satisfy

$$\frac{Mn(\max(w^r, w) - c_1)}{2c_2q} - \phi \sum_{i=1}^N (Q_i w_i^r - D(Q_i)) < 0 \quad (32)$$

$$M > \frac{q\phi(\max(w^r, w) - c_1)}{2} \quad (33)$$

then the minimum area of farmer that choose join the cooperative can get more interests is $S_1 = \frac{\max(w^r, w) - c_1}{2c_2q}$.

Prove: From the above constraint model and Lagrange function, it can be seen that $S_1 = \frac{\max(w^r, w) - c_1}{2c_2q}$ is what we need. But

S_1 must satisfy inequalities (25)(26).For (25),replace the S_1 into it and can obtain the (32).That is

$$\sum_{i=1}^N (Q_i w_i^r - D(Q_i)) > \frac{Mn(\max(w^r, w) - c_1)}{2c_2q\phi} \quad (34)$$

Then,clearly

$$\sum_{i=1}^N (Q_i \max(w_i^r, w_i) - D(Q_i)) > \frac{Mn(\max(w^r, w) - c_1)}{2c_2q\phi} \quad (35)$$

That is

$$-\frac{1}{n}(1-\varphi)\sum_{i=1}^N(Q_i \max(w_i^r, w_i) - D(Q_i)) < -\frac{M(1-\varphi)(\max(w^r, w) - c_1)}{2c_2q\varphi} \quad (36)$$

For (26), the left side is

$$\begin{aligned} & S_1q \max(w^r, w) - C(S_1q) - \frac{1}{n}(1-\varphi)\sum_{i=1}^N(Q_i \max(w_i^r, w_i) - D(Q_i)) - MS_1 - \frac{\sum_{i=1}^N c_{i3}Q_i}{a} \\ & < \frac{(\max(w^r, w) - c_1)^2}{4c_2} - c_0 - \frac{M(1-\varphi)(\max(w^r, w) - c_1)}{2c_2q\varphi} - \frac{M(\max(w^r, w) - c_1)}{2c_2q} - \frac{\sum_{i=1}^N c_{i3}Q_i}{a} \\ & = \frac{(q\phi(\max(w^r, w) - c_1) - 2M)(\max(w^r, w) - c_1)}{4c_2q\phi} - \frac{\sum_{i=1}^N c_{i3}Q_i}{a} - c_0 \end{aligned}$$

According to the inequality (33) and $\max(w^r, w) > c_1, c_{i3} > 0$ which is set before we can get the equation above is negative.

Which satisfy the (26).

3.1.2 The Decision-Making Behavior of Cooperative

For the cooperative, how to make the production plan for each crop before the planting season begins so that it can get the greatest interests? The calculation follows.

Constraint model:

$$\min z = -\pi_e \quad (37)$$

$$s.t. \pi_{f2} - \pi_{f1} < 0, \quad (38)$$

$$-\sum_{i=1}^N c_{i3}Q_i < 0, \quad (39)$$

$z < 0$

Inequality (39) means that the wages of workers are positive.

Substitute the relevant functions:

$$\min z = MS - \varphi \sum_{i=1}^N (Q_i w_i^r - D(Q_i)) \quad (40)$$

$$s.t. Q \max(w^r, w) - C(Q) - \frac{1}{n}(1-\varphi)\sum_{i=1}^N [Q_i \max(w_i^r, w_i) - D(Q_i)] - MS_1 - \frac{\sum_{i=1}^N c_{i3}Q_i}{a} < 0, \quad (41)$$

$$-\sum_{i=1}^N c_{i3}Q_i < 0.$$

Let
$$L(Q_i, \lambda, \mu) = z + \lambda(\pi_{f2} - \pi_{f1}) + \mu \left(-\sum_{i=1}^N c_{i3} Q_i \right) \quad (42)$$

And the relevant KKT conditions are

$$\frac{\partial L(Q_i, \lambda, \mu)}{\partial Q_i} = -\varphi \left(w_i^r - c_{i1} - 2c_{i2} Q_i - c_{i3} \right) + \lambda \left(-\frac{1}{n} (1 - \varphi) \left(\max(w_i^r, w_i) - c_{i1} - c_{i3} - 2c_{i2} Q_i \right) - \frac{c_{i3}}{a} \right) + \mu(-c_{i3}) = 0 \quad (43)$$

$$\frac{\partial L(Q_i, \lambda, \mu)}{\partial c_{i3}} = -\varphi(-Q_i) + \lambda \left(-\frac{1}{n} (1 - \varphi) (-Q_i) - \frac{Q_i}{a} \right) + \mu(-Q_i) = 0 \quad (44)$$

$$\lambda(\pi_{f2} - \pi_{f1}) = 0, \lambda \geq 0 \quad (45)$$

The condition that the cooperative attracts farmers to join is to gain more interests. That is $\pi_{f2} - \pi_{f1} < 0$. So we can get $\lambda = 0$

from the (45). Then substitute it into the equations (43)(44):

$$-\varphi \left(w_i^r - c_{i1} - c_{i3} - 2c_{i2} Q_i \right) + \mu(-c_{i3}) = 0$$

$$-\varphi(-Q_i) + \mu(-Q_i) = 0$$

Solve them and we can obtain

$$\mu = \phi$$

$$Q_i = \frac{w_i^r - c_{i1}}{2c_{i2}} \quad (46)$$

Corollary 2. If the parameters set by the cooperative satisfy

$$\sum_{i=1}^N \left(\frac{(w_i^r - c_{i1} - 2c_{i3})(w_i^r - c_{i1})}{4c_{i2}} - c_{i0} \right) > \frac{MS}{\phi} \quad (47)$$

$$M > \frac{q\phi(\max(w^r, w) - c_1)}{2}$$

Then the optimal production that can not only bring the cooperative the best interests but also ensure that farmers can gain more

incomes by joining the cooperative is
$$Q_i = \frac{w_i^r - c_{i1}}{2c_{i2}} \quad (i = 1 \dots N)$$

Conclusion 1. when $0 < G^{-1}(\eta) \leq w_i^r$, if the parameters set by the cooperative satisfy

$$\sum_{i=1}^N \left[\frac{(w_i^r - c_{i1} - 2c_{i3})(w_i^r - c_{i1})}{4c_{i2}} - c_{i0} \right] > \frac{MS}{\phi}$$

$$M > \frac{q\phi(\max(w^r, w) - c_1)}{2}$$

Then the optimal production of the cooperative is

$$Q_i = \frac{w_i^r - c_{i1}}{2c_{i2}} \quad (i = 1 \dots N)$$

And the minimum area that the farmers choose to join the cooperative can gain more incomes is

$$S_1 = \frac{\max(w^r, w) - c_1}{2c_2q}$$

$$3.2 \quad w_i^r < G^{-1}(\eta) \leq B_i$$

In this case, the revenue function of the cooperative is

$$\pi_e = \phi \sum_{i=1}^N \left[w_i^r Q_i - \frac{1}{\eta} \int_{w_i^r}^{G^{-1}(\eta)} (w_i - w_i^r) Q_i dG(w_i) - D(Q_i) \right] - MS \quad (48)$$

3.2.1 The Decision-Making Behavior of the Farmer

Firstly, set up the constraint model,

$$\min z = MS_1$$

$$s.t. -\pi_e < 0,$$

$$\pi_{f2} - \pi_{f1} < 0.$$

Substitute the functions

$$\min z = MS_1$$

$$s.t. MnS_1 - \phi \sum_{i=1}^N \left[w_i^r Q_i - D(Q_i) - \frac{1}{\eta} \int_{w_i^r}^{G^{-1}(\eta)} (w_i - w_i^r) Q_i dG(w_i) \right] < 0 \quad (49)$$

$$S_1 q \max(w^r, w) - C(S_1 q) - \frac{1}{n} (1 - \phi) \sum_{i=1}^N [Q_i \max(w_i, w_i^r) - D(Q_i)] - MS_1 - \frac{\sum_{i=1}^N c_{i3} Q_i}{a} < 0 \quad (50)$$

Let

$$L(S_1, \lambda, \mu) = z + \lambda(-\pi_e) + \mu(\pi_{f2} - \pi_{f1}).$$

Which is the Lagrange function of the model and the relevant KKT conditions are

$$\frac{\partial L(S_1, \lambda, \mu)}{\partial S_1} = M + \lambda(Mn) + \mu(q \max(w^r, w) - c_1 q - 2c_2 q^2 S_1 - M) = 0 \quad (51)$$

$$\frac{\partial L(S_1, \lambda, \mu)}{\partial M} = S_1 + \lambda(nS_1) + \mu(-S_1) = 0 \quad (52)$$

$$\lambda(-\pi_e) = 0, \lambda \geq 0 \tag{53}$$

Obviously, $-\pi_e < 0$, thus $\lambda = 0, \mu = 1$ and

$$S_1 = \frac{\max(w^r, w) - c_1}{2c_2q} \tag{54}$$

Corollary 3. If the parameters set by the cooperative satisfy

$$\sum_{i=1}^N \left[w_i^r Q_i - D(Q_i) - \frac{1}{\eta} \int_{w_i^r}^{G^{-1}(\eta)} (w_i - w_i^r) Q_i dG(w_i) \right] > \frac{Mn(\max(w^r, w) - c_1)}{2c_2q\phi} \tag{55}$$

$$M > \frac{q\phi(\max(w^r, w) - c_1)}{2}$$

Then the minimum area that the farmers choose to participate in the cooperative can gain more benefits is $S_1 = \frac{\max(w^r, w) - c_1}{2c_2q}$.

3.2.2 The Decision-Making Behavior of the Cooperative

In this case, how to make the production plan that can bring the cooperative the greatest interests? Analysis follows.

Constraint model:

$$\min z = -\pi_e < 0$$

$$s.t. \pi_{f2} - \pi_{f1} < 0$$

$$-\sum_{i=1}^N c_{i3} Q_i < 0$$

Substitute the functions,

$$\min z = MS - \phi \sum_{i=1}^N \left[w_i^r Q_i - D(Q_i) - \frac{1}{\eta} \int_{w_i^r}^{G^{-1}(\eta)} (w_i - w_i^r) Q_i dG(w_i) \right] \tag{56}$$

s.t.

$$Q \max(w^r, w) - C(Q) - \frac{1}{n} (1 - \phi) \sum_{i=1}^N \left[Q_i \max(w_i^r, w_i) - D(Q_i) \right] - MS_1 - \frac{\sum_{i=1}^N c_{i3} Q_i}{a} < 0 \tag{57}$$

$$-\sum_{i=1}^N c_{i3} Q_i < 0$$

$$\text{Let } L(Q_i, \lambda, \mu) = z + \lambda(\pi_{f2} - \pi_{f1}) + \mu \left(-\sum_{i=1}^N c_{i3} Q_i \right).$$

Which is the Lagrange function of the model and the relevant KKT conditions are

$$\frac{\partial L(Q_i, \lambda, \mu)}{\partial Q_i} = -\phi \left(w_i^r - c_{i1} - c_{i3} - 2c_{i2} Q_i - \frac{1}{\eta} \int_{w_i^r}^{G^{-1}(\eta)} (w_i - w_i^r) dG(w_i) \right) +$$

$$\lambda \left(-\frac{1}{n}(1-\phi)(\max(w_i^r, w_i) - c_{i1} - c_{i3} - 2c_{i2}Q_i) - \frac{c_{i3}}{a} \right) + \mu(-c_{i3}) = 0 \quad (58)$$

$$\frac{\partial L(Q_i, \lambda, \mu)}{\partial c_{i3}} = -\phi(-Q_i) + \lambda \left(-\frac{1}{n}(1-\phi)(-Q_i) - \frac{Q_i}{a} \right) + \mu(-Q_i) = 0 \quad (59)$$

$$\lambda(\pi_{f2} - \pi_{f1}) = 0, \lambda \geq 0 \quad (60)$$

The requirement that the cooperative attracts the farmers is the more benefits. So $\pi_{f2} - \pi_{f1} < 0$, then we can get $\lambda = 0$ from equation (60). And $\mu = \phi$

$$Q_i = \frac{w_i^r - c_{i1} - \frac{1}{\eta} \int_{w_i^r}^{G^{-1}(\eta)} (w_i - w_i^r) dG(w_i)}{2c_{i2}} \quad (61)$$

From equations (58)(59).

Corollary 4. If the parameters set by the cooperative satisfy

$$\frac{w_i^r - c_{i1} - P}{2c_{i2}} \left[2(w_i^r - c_{i1} - c_{i3}) - (w_i^r - c_{i1} - P)^2 - 2P \right] - c_{i0} > \frac{MS}{\phi} \quad (62)$$

$$M > \frac{q\phi(\max(w^r, w) - c_1)}{2}$$

Here $P = \frac{1}{\eta} \int_{w_i^r}^{G^{-1}(\eta)} (w_i - w_i^r) dG(w_i)$.

Then the optimal production that can not only bring the cooperative the greatest interests but also ensure the farmers gain more incomes by joining the cooperative is

$$Q_i = \frac{w_i^r - c_{i1} - \frac{1}{\eta} \int_{w_i^r}^{G^{-1}(\eta)} (w_i - w_i^r) dG(w_i)}{2c_{i2}}$$

Conclusion 2. When $w_i^r < G^{-1}(\eta) \leq B_i$, if the parameters set by the cooperative satisfy the inequality (62) and

$$M > \frac{q\phi(\max(w^r, w) - c_1)}{2}, \text{ then the optimal production that can bring the cooperative the greatest benefits is}$$

$$Q_i = \frac{w_i^r - c_{i1} - \frac{1}{\eta} \int_{w_i^r}^{G^{-1}(\eta)} (w_i - w_i^r) dG(w_i)}{2c_{i2}}$$

And the minimum area that the farmers join the cooperative can get more incomes is

$$S_1 = \frac{\max(w^r, w) - c_1}{2c_2q}$$

4. Example

Assume that the cooperative grows three kinds of crops and the relevant parameters are as follows:

crops	w_i	w_i^m	w_i^r	c_{i0}	c_{i1}	c_{i2}	c_{i3}	$G(w_i^r)$
1	$U(0,2)$	1	1.27	300	0.2	0.00004	0.1	0.635
2	$U(1,4)$	2.5	2.3	500	0.4	0.00005	0.1	0.433
3	$U(2,6)$	4	4	700	0.6	0.00006	0.1	0.5

Here the market purchasing price w_i obeys a uniform distribution respectively, and we choose the median w_i^m for comparing.

If $0 < \eta \leq 0.433$, then the optimal production of the cooperative is

$$Q_i = \frac{w_i^r - c_{i1}}{2c_{i2}}$$

The cost function is

$$C(Q_i) = c_{i0} + c_{i1}Q_i + c_{i2}Q_i^2$$

and $D(Q_i) = C(Q_i) + c_{i3}Q_i$. Then substitute the data.

Crops	1	2	3
Q_i	13375	19000	28333
$D(Q_i)$	11468	28050	68698
$Q_i w_i^r$	16986	43700	113332
$Q_i \max(w_i^r, w_i)$	16986	47500	113332

Assume the individual farmer without the cooperative plants the crop 1 and the relevant parameters are as follows.

w	w^m	w^r	c_0	c_1	c_2
$U(0,2)$	1	1.27	300	0.2	0.00005

The cost function of the individual farmer is $C(Q) = c_0 + c_1Q + c_2Q^2$. And the c_2 should be larger than c_{12} because of the specialization and scale of the cooperative.

Assume that the land area of the farmer satisfy

$$S_1 = \frac{\max(w^r, w) - c_1}{2c_2q}$$

Then for an individual farmer, the production is

$$Q = S_1q = \frac{\max(w^r, w) - c_1}{2c_2} = 10700 \tag{63}$$

And the revenue is

$$\pi_{f2} = Q \max(w^r, w) - C(Q) = 5424.5 \tag{64}$$

For a farmer that join the cooperative, firstly the fixed incomes that the cooperative provides is

$$MS_1 = \frac{q\varphi(\max(w^r, w) - c_1)}{2} \cdot \frac{\max(w^r, w) - c_1}{2c_2q} = \frac{\varphi(\max(w^r, w) - c_1)^2}{4c_2} = 5724.5\varphi \tag{65}$$

Obviously, the fixed incomes must satisfy $MS_1 < \pi_{f2}$. That is

$$\begin{aligned} \frac{\varphi(\max(w^r, w) - c_1)^2}{4c_2} &< (\max(w^r, w) - c_1 - c_2Q)Q - c_0 \\ \varphi &< \frac{4(\max(w^r, w) - c_1 - c_2Q)Qc_2 - 4c_2c_0}{(\max(w^r, w) - c_1)^2} = 0.95 \end{aligned} \tag{66}$$

Assume that the total number of workers $a = 8$ and the proportion of the land area $n = 10$. Then the revenue of the farmer that joins the cooperative is

$$\pi_{f1} = \frac{1}{n}(1 - \varphi) \sum_{i=1}^3 (Q_i \max(w_i^r, w_i) - D(Q_i)) + MS_1 + \frac{\sum_{i=1}^3 c_i Q_i}{a} = 7718.2 - 1235.7\varphi \tag{67}$$

If φ satisfy the inequality (66), then clearly

$$7718.2 - 1235.7\varphi > 5424.5$$

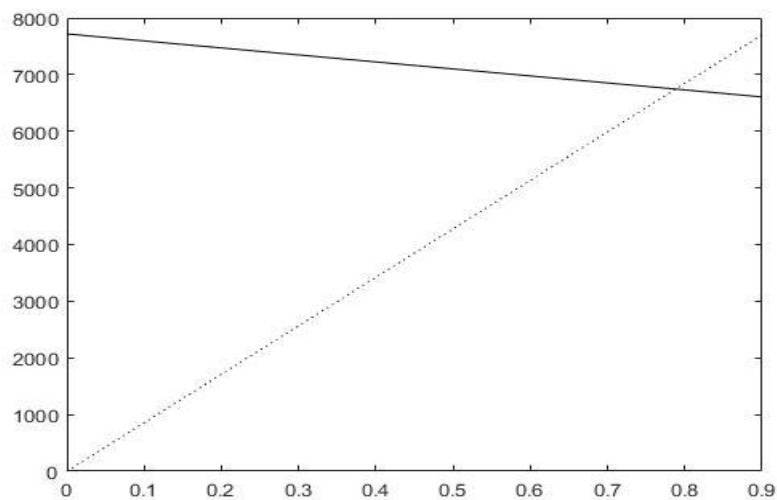
That is $\pi_{f1} > \pi_{f2}$. In this case, the changing image that π_{f1} about φ is shown by the real line in the figure below. Obviously,

the larger the φ is, the smaller π_{f1} is, but will not less than $\pi_{f2} = 5424.5$. It follows that the cooperative can ensure the interests of the farmers and reduce the risk.

And the revenue of the cooperative is

$$\pi_e = \varphi \sum_{i=1}^3 (Q_i w_i^r - D(Q_i)) - MnS_1 = 8857\varphi \tag{68}$$

The changing image that π_e about φ is shown by the broken line in the following figure. Clearly, the larger φ is, the greater the profit π_e . So the cooperative and the farmers must define φ before signing the contract.



The scale of the cooperative in the example here is small and the benefit is low. In the actual operation, the cooperative can expand the scale to attract more farmers and increase profits.

5. Conclusion

This paper constructs a three level order agricultural supply chain model of “farmers+cooperative+company”, and obtain the specific benefits of the cooperative under different risk-aversion by the CVaR measure, then set up the relevant constraint model and analyses the different decision-making behaviors of the cooperative and farmers by the Lagrange function. It follows that the minimum area that the farmers can gain more benefits by joining the cooperative and the minimum fixed incomes that provided by the cooperative to ensure the farmers’ interests.

It can be seen that the establishment of the cooperative can not only guarantee their own interests, but also ensure the benefits of the farmers. Which can improve the contract signing rate and the order fulfillment rate of the farmers and provide a solution for the low performance rate in the order agriculture.

With the rapid development of modern society, many young people choose to go out and start their own businesses, which leads to the increasing number of left behind elderly and children in rural areas. Most of them are weak in labor capacity, so the establishment of the cooperative is very important. Thus, the cooperative mode of order agriculture will become more and more mainstream, and this article provides a theoretical basis for the establishment of the cooperative, which will further develop the scale and unification of agricultural production in China. But this article is in the perfect state of analysis, in fact, there are many uncertainties, such as the specific game behavior between the cooperative and

companies, competition between cooperatives and cooperatives, which will have impacts on cooperatives’ decisions. These problems can be further studied to make the cooperative closer to life.

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