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Controllability of Stochastic Impulsive Quasilinear Neutral Integrodifferential Systems

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Abstract

This paper derives the sufficient conditions for controllability of stochastic impulsive quasilinear neutral integrodifferential systems with time varying delays in Hilbert spaces. The results are obtained by using semigroup theory, evolution operator and fixed point technique. An example is provided to illustrate the obtained results.

Keywords: Controllability, Stochastic Impulsive quasilinear neutral integrodifferential systems, Time varying delays, Fixed point.

2015 Mathematics Subject Classification: 93B05, 34A37, 34K50.

1 Introduction

The purpose of this paper is to study the controllability of stochastic impulsive quasilinear neutral integrodifferential time varying delay systems with nonlocal conditions which is of the form

$$\begin{aligned} d\Big[x(t) - g\Big(t, x(t), x(\alpha_1(t)), \cdots, x(\alpha_m(t))\Big)\Big] \\ = \Big[A(t,x)x(t) + f\Big(t, x(t), x(\beta_1(t)), \cdots, x(\beta_{n-1}(t)), \int_0^t h\big(t, s, x(\beta_n(s))\big)ds\Big) \\ + Bu(t)\Big]dt + \sigma\Big(t, x(t), x(\gamma_1(t)), \cdots, x(\gamma_q(t))\Big)dw(t), \quad t \in J := [0,a], t \neq \tau_k, \\ \triangle x(\tau_k) \;=\; x(\tau_k^+) - x(\tau_k^-) = I_k(x(\tau_k^-)), \qquad k = 1, 2, \cdots, m, \\ x(0) \;+\; H(t_1, t_2, \cdots, t_p, x(t_1), x(t_2), \cdots, x(t_p)) = x_0. \end{aligned} \tag{1.1}$$

where $0 < t_1 < t_2 < \cdots < t_p \leq a$ $(p \in N)$. Here, $A(t,x)$ is the infinitesimal generator of a C_0- semigroup in H and B is a bounded linear operator from U into H. The state variable $x(\cdot)$ takes values in a real separable Hilbert space H with innerproduct $(\cdot,\cdot)$ and norm $\|\cdot\|$ and the control function $u(\cdot)$ takes values in $L^2(J,U)$, a Banach space of admissible control functions for a separable Hilbert space U. Let K be another separable Hilbert space with inner product $(\cdot,\cdot)_K$ and $\|\cdot\|_K$. Suppose $\{w(t)\}_{t\geq 0}$ is a given K-valued Wiener process with a finite trace covariance operator $Q \geq 0$. We use the same notation $\|\cdot\|$ for the norm $\mathcal{L}(K,H)$, where $\mathcal{L}(K,H)$ denotes the space of all bounded linear operators from K into H. Further $g : J \times H^{m+1} \to H$, $f : J \times H^{n+1} \to H$, $h : \Lambda \times H \to H$, $\sigma : J \times H^{q+1} \to \mathcal{L}_Q(K,H)$ are measurable mappings in H-norm and $\mathcal{L}_Q(K,H)$ norm respectively, where $\mathcal{L}_Q(K,H)$ denotes

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the space of all Q-Hilbert-Schmidt operators from K into H will be defined in Section 2 and $\Lambda = \{(t,s) \in J \times J : s \leq t\}$. The delays $\alpha_i(t)$, $\beta_j(t)$, $\gamma_\kappa(t)$ are continuous scalar valued functions defined on J such that $\alpha_i(t) \leq t$, $\beta_j \leq t$, $\gamma_\kappa \leq t$ for $i = 1,2,\cdots,m$, $j = 1,2,\cdots,n$, $\kappa = 1,2,\cdots,q$. Here, the nonlocal function $H : \mathcal{PC}[J^p \times H^p : H] \to H$ and the impulsive function $I_k \in C(H,H)$ $(k = 1,2,\cdots,m)$ are bounded functions. Furthermore, the fixed times τ_k satisfies $0 = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_m < a$, $x(\tau_k^+)$ and $x(\tau_k^-)$ denote the right and left limits of $x(t)$ at $t = \tau_k$. And $\triangle x(\tau_k) = x(\tau_k^+) - x(\tau_k^-)$ represents the jump in the state x at time τ_k, where I_k determines the size of the jump.

Neutral delay differential equations (NDDEs) are often used to describe the dynamical systems which not only depend on present and past states but also involve derivatives with delays. Practical examples of neutral delay differential systems include the distributed networks containing lossless transmission lines [18], population ecology [18], processes including steam or water pipes, heat exchanges [17] and other engineering systems [17]. NDDEs are considered as a branch of delay differential equations (DDEs). DDEs can provide us a realistic model of many phenomena arising in several areas of applied mathematics. For instance, infectious diseases, population dynamics, physiological and pharmaceutical kinetics and chemical kinetics, the navigational control of ships and aircrafts and control problems.

DDEs have been used for many years in control theory and only recently have been applied to biological models. In biological and mechanical processes we find often physical delays. Delay equations are used to make the mathematical model closer to the real phenomenon. Some examples of delay mathematical models in biology are population dynamics, ecology, epidemiology, immunology, physiology, neurology. Hence the theory of neutral delay differential equations is even more complicated than the theory of non-neutral delay equations. In the last few decades, there has been increasing interest in the study of delay differential equations and neutral delay differential equations for several investigators. The dynamics of many evolving processes are subject to abrupt changes, such as shocks, harvesting and natural disasters. These phenomena involve short-term perturbations, whose duration is negligible in comparison with the duration of an entire evolution. Systems with short-term perturbations are often naturally described by impulsive differential equations (for example, see [20, 26]). Stochastic differential and integrodifferential equations have attracted great attention due to their applications in characterizing many problems in physics, biology, mechanics and so on.

Quasilinear evolution equations are encountered in many areas of science and engineering. Several authors have discussed the existence of solutions of abstract quasilinear evolution equations in Banach spaces [1, 12, 15, 16, 23, 24]. Recently, the study on controllability of quasilinear systems has gained renewed interests and only few papers have appeared (see [3, 4, 6, 8]). The qualitative properties such as existence, uniqueness and regularity of solutions of functional and neutral functional differential equations with nonlocal conditions have been studied by some researchers [2, 13, 14]. Lin and Liu [21] discussed the semilinear integrodifferential equations with nonlocal Cauchy problem. Balachandran

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et al. [5] investigated the existence results for abstract neutral differential equations with time varying delays. The nonlocal Cauchy problem for delay integrodifferential equations of Sobolev type in Banach spaces was discussed by [7]. Balasubramaniam et al. [9] derived the existence of solutions for nonlocal neutral stochastic functional differential equations. Stochastic controllability and approximate controllability of nonlinear stochastic systems with multiple delays and timevarying delays were studied by [19, 22]. Subalakshmi and Balachandran [27] discussed the approximate controllability of nonlinear stochastic impulsive integrodifferential systems in Hilbert spaces. Upto now, there is no work reported on the controllability of stochastic impulsive quasilinear neutral integrodifferential timevarying delay systems and this fact is the motivation of our present work. In this paper, we make an attempt to fill this gap by studying the controllability of the system (1.1).

2 Preliminaries

Let $(\Omega, \mathcal{F}, P; \mathbf{F})$ $\{\mathbf{F} = \{\mathcal{F}_t\}_{t\geq 0}\}$ be a complete filtered probability space satisfying that $\mathcal{F}_0$ contains all P-null sets of $\mathcal{F}$. An H-valued random variable is an $\mathcal{F}$-measurable function $x(t)$: $\Omega \to H$ and a collection of random variables $S = \{x(t,\omega) : \Omega \to H \setminus t \in J\}$ is called a stochastic process. Usually, we suppress the dependence on $\omega \in \Omega$ and write $x(t)$ instead of $x(t,\omega)$ and $x(t) : J \to H$ in the place of S.

Let $\{e_n\}_{n=1}^{\infty}$ be a complete orthonormal basis of K. Suppose that $\{w(t) : t \geq 0\}$ is a cylindrical K-valued wiener process with a finite trace nuclear covariance operator $Q \geq 0$, denote $Tr(Q) = \sum_{n=1}^{\infty}\lambda_n = \lambda < \infty$, which satisfies that $Qe_n = \lambda_n e_n$. So, actually, $w(t) = \sum_{n=1}^{\infty} \sqrt{\lambda_n}\omega_n(t)e_n$, where $\{\omega_n(t)\}_{n=1}^{\infty}$ are mutually independent one-dimensional standard Wiener processes. We assume that $\mathcal{F}_t = \sigma\{w(s) : 0 \leq s \leq t\}$ is the σ-algebra generated by w and $\mathcal{F}_a = \mathcal{F}$. Let $\Psi \in \mathcal{L}(K,H)$ and define

$$\|\Psi\|_Q^2 = Tr(\Psi Q \Psi^*) = \sum_{n=1}^{\infty} \|\sqrt{\lambda_n}\Psi e_n\|^2.$$

If $\|\Psi\|_Q < \infty$, then Ψ is called a Q-Hilbert-Schmidt operator. Let $\mathcal{L}_Q(K,H)$ denote the space of all Q-Hilbert-Schmidt operators $\Psi : K \to H$. The completion $\mathcal{L}_Q(K,H)$ of $\mathcal{L}(K,H)$ with respect to the topology induced by the norm $\|\cdot\|_Q$ where $\|\Psi\|_Q^2 = \langle\Psi,\Psi\rangle$ is a Hilbert space with the above norm topology. For more details in this section refer Prato [10].

The collection of all strongly measurable, square integrable H-valued random variables denoted by $\mathcal{L}_2(\Omega,\mathcal{F},P;H) \equiv \mathcal{L}_2(\Omega,H)$, is a Banach space equipped with norm $\|x(\cdot)\|_{\mathcal{L}_2} = (E\|x(\cdot;\omega)\|_H^2)^{\frac{1}{2}}$, where the expectation E is defined by $E(h) = \int_\Omega h(\omega)dP$. Similarly, $\mathcal{L}_2^{\mathcal{F}}(\Omega,H)$ denotes the Banach space of all $\mathcal{F}_t$-measurable, square integrable random variables, such that $\int_\Omega \|x(t,\cdot)\|_{\mathcal{L}_2}^2 dt < \infty$. Denote $J_0 = [0,\tau_1], J_k = (\tau_k,\tau_{k+1}], k = 1,2,\cdots,m$, and define the following class of functions:
$\mathcal{PC}(J,\mathcal{L}_2(\Omega,H))$={$x : J \to \mathcal{L}_2$: x(t) is continuous everywhere except for some τ_k at which $x(\tau_k^-)$ and $x(\tau_k^+)$ exists and $x(\tau_k^-) = x(\tau_k), k = 1,2,3,\cdots,m$} is the Banach space of piecewise continuous maps from J into $\mathcal{L}_2(\Omega,H)$ satisfying the condition $\sup_{t\in J}E\|x(t)\|^2 < \infty$.

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Let $\mathcal{H}_2 \equiv \mathcal{PC}(J, \mathcal{L}_2)$ be the closed subspace of $\mathcal{PC}(J, \mathcal{L}_2^{\mathcal{F}}(\Omega, H))$ consisting of measurable, $\mathcal{F}_t$-adapted and H-valued processes $x(t)$. Then $\mathcal{PC}(J, \mathcal{L}_2)$ is a Banach space endowed with the norm

$$\|x\|_{\mathcal{PC}}^2 = \sup_{t \in J} \ \{E\|x(t)\|^2 : x \in \mathcal{PC}(J, \mathcal{L}_2)\}.$$

Let H and Y be two Hilbert spaces such that Y is densely and continuously embedded in H. For any Hilbert space $\mathcal{Z}$ the norm of $\mathcal{Z}$ is denoted by $\|\cdot\|_{\mathcal{Z}}$ or $\|\cdot\|$. The space of all bounded linear operators from H to Y is denoted by $B(H, Y)$ and $B(H, H)$ is written as $B(H)$. We recall some definitions and known facts from [25].

Definition: 2.1 *Let S be a linear operator in H and let Y be a subspace of H. The operator $\tilde{S}$ defined by $D(\tilde{S}) = \{x \in D(S) \cap Y : Sx \in Y\}$ and $\tilde{S}x = Sx$ for $x \in D(\tilde{S})$ is called the part of S in Y.*

Definition: 2.2 *Let Q be a subset of H and for every $0 \le t \le a$ and $b \in Q$, let $A(t, b)$ be the infinitesimal generator of a C_0 semigroup $S_{t,b}(s), s \ge 0$ on H. The family of operators $\{A(t, b)\}$, $(t, b) \in J \times Q$, is stable if there are constants $M \ge 1$ and ω such that*

$$\begin{aligned} \rho(A(t,b)) &\supset (\omega, \infty) \qquad for\ (t,b) \in J \times Q, \\ \Big\| \prod_{j=1}^{k} R(\lambda : A(t_j, b_j)) \Big\| &\le M(\lambda - \omega)^{-k} \quad for \ \ \lambda > \omega \end{aligned}$$

and every finite sequences $0 \le t_1 \le t_2 \le \cdots \le t_k \le a$, $b_j \in Q, 1 \le j \le k$. The stability of $\{A(t, b)\}$, $(t, b) \in J \times Q$, implies [25] that

$$\Big\| \prod_{j=1}^{k} S_{t_j, b_j}(s_j) \Big\| \le M \exp\ \{\omega \sum_{j=1}^{k} s_j\} \quad for\ s_j \ge 0$$

and any finite sequences $0 \le t_1 \le t_2 \le \cdots \le t_k \le a$, $b_j \in Q, 1 \le j \le k$. $k = 1, 2, \cdots$.

Definition: 2.3 *Let $S_{t,b}(s)$, $s \ge 0$ be the C_0 semigroup generated by $A(t, b), (t, b) \in J \times Q$. A subspace Y of H is called $A(t, b)$- admissible if Y is invariant subspace of $S_{t,b}(s)$ and the restriction of $S_{t,b}(s)$ to Y is a C_0- semigroup in Y.*

Let $Q \subset H$ be a subset of H such that for every $(t, b) \in J \times Q$, $A(t, b)$ is the infinitesimal generator of a C_0-semigroup $S_{t,b}(s), s \ge 0$ on H. We make the following assumptions:

(E1) The family $\{A(t, b)\}, (t, b) \in J \times Q$ is stable.

(E2) Y is $A(t, b)$- admissible for $(t, b) \in J \times Q$ and the family $\{\tilde{A}(t, b)\}, (t, b) \in J \times Q$ of parts $\tilde{A}(t, b)$ of $A(t, b)$ in Y, is stable in Y.

(E3) For $(t, b) \in J \times Q$, $D(A(t, b)) \supset Y$, $A(t, b)$ is a bounded linear operator from Y to H and $t \to A(t, b)$ is continuous in the $B(Y, H)$ norm $\|\cdot\|$ for every $b \in Q$.

Research & Analysis Journals

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(E4) There is a constant $L > 0$ such that

$$\|A(t,b_1) - A(t,b_2)\|_{Y\to H} \le L\|b_1 - b_2\|_H$$

holds for every $b_1, b_2 \in Q$ and $0 \le t \le a$.

Let Q be a subset of H and let $\{A(t,b)\}$, $(t,b) \in J\times Q$ be a family of operators satisfying the conditions $(E1) - (E4)$. If $x \in \mathcal{PC}(J,\mathcal{L}_2)$ has values in Q then there is a unique evolution system $U(t,s;x), 0 \le s \le t \le a$ in H satisfying (see [25])

(i) $\|U(t,s;x)\| \le Me^{\omega(t-s)}$ for $0 \le s \le t \le a$, where M and ω are stability constants.

(ii) $\frac{\partial^+}{\partial t}U(t,s;x)y = A(s,x(s))U(t,s;x)y$ for $y \in Y,\ 0 \le s \le t \le a$.

(iii) $\frac{\partial}{\partial s}U(t,s;x)y = -U(t,s;x)A(s,x(s))y$ for $y \in Y,\ 0 \le s \le t \le a$.

Definition: 2.4 [11] *A stochastic process x is said to be a mild solution of (1.1) if the following conditions are satisfied:*

(a) $x(t,\omega)$ is a measurable function from $J\times\Omega$ to H and $x(t)$ is $\mathcal{F}_t$ -adapted,

(b) $E\|x(t)\|^2 < \infty$ for each $t \in J$,

(c) $\triangle x(\tau_k) = x(\tau_k^+) - x(\tau_k^-) = I_k(x(\tau_k^-)),\ k = 1,2,\cdots,m$,

(d) For each $u \in L_2^{\mathcal{F}}(J,U)$, the process x satisfies the following integral equation

$$\begin{aligned} x(t) = {} & U(t,0;x)\big[x_0 - H(t_1,\cdots,t_p,x(t_1),\cdots,x(t_p)) - g\big(0,x(0),x(\alpha_1(0)),\cdots,x(\alpha_m(0))\big)\big] \\ & + g\big(t,x(t),x(\alpha_1(t)),\cdots,x(\alpha_m(t))\big) + \int_0^t U(t,s;x)Bu(s)ds \\ & + \int_0^t U(t,s;x)A(s,x(s))g\big(s,x(s),x(\alpha_1(s)),\cdots,x(\alpha_m(s))\big)ds \\ & + \int_0^t U(t,s;x)f\Big(s,x(s),x(\beta_1(s)),\cdots,x(\beta_{n-1}(s)),\int_0^s h\big(s,\eta,x(\beta_n(\eta))\big)d\eta\Big)ds \\ & + \int_0^t U(t,s;x)\sigma\Big(s,x(s),x(\gamma_1(s)),\cdots,x(\gamma_q(s))\Big)dw(s) \\ & + \sum_{0<\tau_k<t} U(t,\tau_k;x)I_k(x(\tau_k^-)), \quad \text{for a.e. } t \in J, \\ x(0) \ + \ & H(t_1,t_2,\cdots,t_p,x(t_1),x(t_2),\cdots,x(t_p)) = x_0 \in H. \end{aligned} \tag{2.1}$$

Definition: 2.5 *The system (1.1) is said to be controllable on the interval J, if for every initial condition x_0 and $x_1 \in H$, there exists a control $u \in L^2(J,U)$ such that the solution $x(\cdot)$ of (1.1) satisfies $x(a) = x_1$.*

Further there exists a constant $L_0 > 0$ such that for every $x, y \in \mathcal{H}_2$ and every $\tilde{y} \in Y$ we have

$$\|U(t,s;x)\tilde{y} - U(t,s;y)\tilde{y}\|^2 \le L_0a^2\|\tilde{y}\|_Y^2\|x-y\|_{\mathcal{PC}}^2.$$

In order to establish our controllability result we assume the following hypotheses:

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(H1) $A(t,x)$ generates a family of evolution operators $U(t,s;x)$ in H and there exists a constant $L_U > 0$ such that

$$\|U(t,s;x)\|^2 \leq L_U \quad \text{for } 0 \leq s \leq t \leq a,\ x \in \mathcal{H}_2.$$

(H2) The linear operator $W : L^2(J,U) \to H$ defined by

$$Wu = \int_0^a U(a,s;x)Bu(s)ds$$

is invertible with inverse operator W^{-1} taking values in $L^2(J,U) \setminus kerW$ and there exists a positive constant L_W such that

$$\|BW^{-1}\|^2 \leq L_W.$$

From $(H3)$-$(H7)$, let $\mathcal{Z}$ be taken as both H and Y.

(H3) (i) The function $g : J \times \mathcal{Z}^{m+1} \to \mathcal{Z}$ is continuous and there exist constants $L_g > 0$, $\tilde{L_g} > 0$ for $t,s \in J$ and $x_i, y_i \in \mathcal{Z}$, $i = 1,2,\cdots,m+1$ such that the function $A(t,x)g$ satisfies the Lipschitz condition:

$$\begin{aligned} E\|A(t,x(t))\ g(t,x_1,x_2,\cdots,x_{m+1}) \ &- \ A(t,y(t))\ g(t,y_1,y_2,\cdots,y_{m+1})\|^2 \\ &\leq \ L_g\Big[\sum_{i=1}^{m+1}\|x_i-y_i\|^2\Big], \end{aligned}$$

and $\tilde{L}_g = \sup_{t\in J}\|A(t,0)g(t,0,\cdots,0)\|^2$.

(ii) There exist constants $L_1 > 0, L_2 > 0$ and $L_3 > 0$ such that

$$\begin{aligned} E\|g(t,x_1,x_2,\cdots,x_{m+1}) \ &- \ g(s,y_1,y_2,\cdots,y_{m+1})\|^2 \\ &\leq \ L_1\Big[|t-s|^2 + \sum_{i=1}^{m+1}\|x_i-y_i\|^2\Big] \text{ and} \\ E\|g(t,x_1,x_2,\cdots,x_{m+1})\|^2 \ &\leq \ L_2\sum_{i=1}^{m+1}\|x_i\|^2 + L_3. \end{aligned}$$

(H4) The nonlinear function $f : J \times \mathcal{Z}^{n+1} \to \mathcal{Z}$ is continuous and there exist constants $L_f > 0$, $\tilde{L_f} > 0$ for $t \in J$ and $x_j, y_j \in \mathcal{Z}$, $j = 1,2,\cdots,n+1$ such that

$$E\|f(t,x_1,x_2,\cdots,x_{n+1}) - f(t,y_1,y_2,\cdots,y_{n+1})\|^2 \leq L_f\Big[\sum_{j=1}^{n+1}\|x_j-y_j\|^2\Big]$$

and $\tilde{L}_f = \sup_{t\in J}\|f(t,0,\cdots,0)\|^2$.

(H5) The nonlinear function $h : \Lambda \times \mathcal{Z} \to \mathcal{Z}$ is continuous and there exist positive constants L_h, $\tilde{L}_h$, for $x,y \in \mathcal{Z}$ and $(t,s) \in \Lambda$ such that

$$E\Big\|\int_0^t \Big[h(t,s,x) - h(t,s,y)\Big]ds\Big\|^2 \leq L_h\|x-y\|^2$$

and $\tilde{L}_h = \sup_{(t,s)\in\Lambda}\|\int_0^t h(t,s,0)ds\|^2$.

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(H6) The function $\sigma : J\times\mathcal{Z}^{q+1} \to \mathcal{L}_Q(K,H)$ is continuous and there exist constants $L_\sigma > 0$, $\tilde{L}_\sigma > 0$ for $t \in J$ and $x_\kappa, y_\kappa \in \mathcal{Z}$, $\kappa = 1,2,\cdots,q+1$ such that

$$E\|\sigma(t,x_1,x_2,\cdots,x_{q+1}) - \sigma(t,y_1,y_2,\cdots,y_{q+1})\|_Q^2 \le L_\sigma\Big[\sum_{\kappa=1}^{q+1}\|x_\kappa - y_\kappa\|^2\Big]$$

and $\tilde{L}_\sigma = \sup_{t\in J}\|\sigma(t,0,\cdots,0)\|^2$.

(H7) The nonlocal function $H : \mathcal{PC}(J^p \times \mathcal{Z}^p : \mathcal{Z}) \to \mathcal{Z}$ is continuous and there exist constants $L_H > 0$, $\tilde{L}_H > 0$ such that

$$E\|H(t_1,\cdots,t_p,x(t_1),\cdots,x(t_p)) - H(t_1,\cdots,t_p,y(t_1),\cdots,y(t_p))\|^2 \le L_H\|x-y\|^2,$$

$$E\|H(t_1,t_2,\cdots,t_p,x(t_1),x(t_2),\cdots,x(t_p))\|^2 \le \tilde{L}_H.$$

(H8) $I_k : \mathcal{H}_2 \to \mathcal{H}_2$ is continuous and there exist constants $\beta_k > 0$, $\tilde{\beta}_k > 0$ for $x,y \in \mathcal{Z}$ such that

$$E\|I_k(x) - I_k(y)\|^2 \le \beta_k\|x-y\|^2, \quad k = 1,2,\cdots,m$$

and $\tilde{\beta}_k = \|I_k(0)\|^2$, $k = 1,2,\cdots,m$.

(H9) There exists a constant $r > 0$ such that

$$9\Big\{L_U(\|x_0\|^2 + \tilde{L}_H) + L_U[2L_2(m+1)(\|x_0\|^2 + \tilde{L}_H) + L_3] + L_2(m+1)r + L_3 + 2a^2L_U[L_g(m+1)r + \tilde{L}_g] + a^2L_U\mathcal{G} + 2a^2L_U[L_f((n+2L_h)r + 2\tilde{L}_h) + \tilde{L}_f] + 2aL_UTr(Q)[L_\sigma(q+1)r + \tilde{L}_\sigma] + 2mL_U\Big[\sum_{k=1}^m\beta_k r + \sum_{k=1}^m\tilde{\beta}_k\Big]\Big\} \le r \text{ and}$$

$$\nu = 9\Big\{(1+16a^2L_UL_W)(N_1+N_2+N_3+N_4+N_5+N_6) + 2a^3L_0\mathcal{G}\Big\}$$

where

$$N_1 = L_0a^2\|x_0\|^2 + 2\big[L_0a^2\tilde{L}_H + L_UL_H\big]$$

$$N_2 = 2\Big[L_0a^2\big[2L_2(m+1)(\|x_0\|^2+\tilde{L}_H)+L_3\big] + L_UL_1(m+1)L_H\Big] + L_1(m+1),$$

$$N_3 = 2a^2\Big[2aL_0\big(L_g(m+1)r + \tilde{L}_g\big) + L_UL_g(m+1)\Big],$$

$$N_4 = 2a^2\Big[2aL_0\big(L_f((n+2L_h)r + 2\tilde{L}_h) + \tilde{L}_f\big) + L_UL_f(n+L_h)\Big],$$

$$N_5 = 2a\Big[2aL_0Tr(Q)\big(L_\sigma(q+1)r + \tilde{L}_\sigma\big) + L_UTr(Q)(q+1)L_\sigma\Big],$$

$$N_6 = 2m\left[2a^2L_0\Big(\sum_{k=1}^m\beta_k r + \sum_{k=1}^m\tilde{\beta}_k\Big) + L_U\sum_{k=1}^m\beta_k\right].$$

3 Controllability Result

Theorem: 3.1 *If the conditions $(H1)-(H9)$ are satisfied and if $0 \le \nu < 1$, then the system (1.1) is controllable on J.*

Research & Analysis Journals

RA Journal of Applied Research

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Proof: Using the hypothesis $(H2)$ for an arbitrary function $x(\cdot)$, define the control

$$\begin{aligned} u(t) = W^{-1}\Big[&x_1 - U(a,0;x)\big[x_0 - H(t_1,\cdots,t_p,x(t_1),\cdots,x(t_p)) - g\big(0,x(0),\cdots,x(\alpha_m(0))\big)\big] \\ &-g\big(a,x(a),x(\alpha_1(a)),\cdots,x(\alpha_m(a))\big) - \sum_{0<\tau_k<a} U(a,\tau_k;x)I_k(x(\tau_k^-)) \\ &-\int_0^a U(a,s;x)A(s,x(s))g\big(s,x(s),x(\alpha_1(s)),\cdots,x(\alpha_m(s))\big)ds \\ &-\int_0^a U(a,s;x)f\Big(s,x(s),x(\beta_1(s)),\cdots,x(\beta_{n-1}(s)),\int_0^s h\big(s,\eta,x(\beta_n(\eta))\big)d\eta\Big)ds \\ &-\int_0^a U(a,s;x)\sigma\Big(s,x(s),x(\gamma_1(s)),\cdots,x(\gamma_q(s))\Big)dw(s)\Big](t). \end{aligned} \tag{3.1}$$

Let $\mathcal{Y}_r$ be a nonempty closed subset of $\mathcal{H}_2$ defined by

$$\mathcal{Y}_r = \{x : x \in \mathcal{H}_2 | E\|x(t)\|^2 \le r\}.$$

Consider a mapping $\Phi : \mathcal{Y}_r \to \mathcal{Y}_r$ defined by

$$\begin{aligned} (\Phi x)(t) &= U(t,0;x)\big[x_0 - H(t_1,\cdots,t_p,x(t_1),\cdots,x(t_p)) - g\big(0,x(0),x(\alpha_1(0)),\cdots,x(\alpha_m(0))\big)\big] \\ &+g\big(t,x(t),x(\alpha_1(t)),\cdots,x(\alpha_m(t))\big) + \int_0^t U(t,s;x)Bu(s)ds \\ &+\int_0^t U(t,s;x)A(s,x(s))g\big(s,x(s),x(\alpha_1(s)),\cdots,x(\alpha_m(s))\big)ds \\ &+\int_0^t U(t,s;x)f\Big(s,x(s),x(\beta_1(s)),\cdots,x(\beta_{n-1}(s)),\int_0^s h\big(s,\eta,x(\beta_n(\eta))\big)d\eta\Big)ds \\ &+\int_0^t U(t,s;x)\sigma\Big(s,x(s),x(\gamma_1(s)),\cdots,x(\gamma_q(s))\Big)dw(s) + \sum_{0<\tau_k<t} U(t,\tau_k;x)I_k(x(\tau_k^-)), \end{aligned}$$

We have to show that by using the above control the operator Φ has a fixed point. Since all the functions involved in the operator are continuous therefore Φ is continuous. For convenience let us take

$$\begin{aligned} V(\mu,x) = BW^{-1}\Big[&x_1 - U(a,0;x)\big[x_0 - H(t_1,\cdots,t_p,x(t_1),\cdots,x(t_p)) - g\big(0,x(0),\cdots,x(\alpha_m(0))\big)\big] \\ &-g\big(a,x(a),x(\alpha_1(a)),\cdots,x(\alpha_m(a))\big) - \sum_{0<\tau_k<a} U(a,\tau_k;x)I_k(x(\tau_k^-)) \\ &-\int_0^a U(a,s;x)A(s,x(s))g\big(s,x(s),x(\alpha_1(s)),\cdots,x(\alpha_m(s))\big)ds \\ &-\int_0^a U(a,s;x)f\Big(s,x(s),x(\beta_1(s)),\cdots,x(\beta_{n-1}(s)),\int_0^s h\big(s,\eta,x(\beta_n(\eta))\big)d\eta\Big)ds \\ &-\int_0^a U(a,s;x)\sigma\Big(s,x(s),x(\gamma_1(s)),\cdots,x(\gamma_q(s))\Big)dw(s)\Big](\mu). \end{aligned}$$

From our assumptions we have

$$\begin{aligned} E\|V(\mu,x)\|^2 \le\ & 9L_W\Big\{\|x_1\|^2 + L_U(\|x_0\|^2 + \tilde{L}_H) + L_U[2L_2(m+1)(\|x_0\|^2 + \tilde{L}_H) + L_3] \\ &+L_2(m+1)r + L_3 + 2a^2L_U[L_g(m+1)r + \tilde{L}_g] + 2a^2L_U[L_f((n+2L_h)r + 2\tilde{L}_h) \\ &+\tilde{L}_f] + 2aL_UTr(Q)[L_\sigma(q+1)r + \tilde{L}_\sigma] + 2mL_U\Big[\sum_{k=1}^m \beta_k r + \sum_{k=1}^m \tilde{\beta}_k\Big]\Big\} := \mathcal{G}. \end{aligned}$$

RA Journal of Applied Research
||Volume||3||Issue||01||Pages-822-833||Jan-2017|| ISSN (e): 2394-6709
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and

$$\begin{aligned} E\|V(\mu,x)-V(\mu,y)\|^2 &\leq 8L_W\Big\{L_0a^2\|x_0\|^2+2\big[L_0a^2\tilde{L}_H+L_UL_H\big]+L_1(m+1)\\ &+2\Big[L_0a^2\big[2L_2(m+1)(\|x_0\|^2+\tilde{L}_H)+L_3\big]+L_UL_1(m+1)L_H\Big]\\ &+2a^2\Big[2aL_0\big(L_g(m+1)r+\tilde{L}_g\big)+L_UL_g(m+1)\Big]\\ &+2a^2\Big[2aL_0\big(L_f((n+2L_h)r+2\tilde{L}_h)+\tilde{L}_f\big)+L_UL_f(n+L_h)\Big]\\ &+2a\Big[2aL_0Tr(Q)\big(L_\sigma(q+1)r+\tilde{L}_\sigma\big)+L_UTr(Q)(q+1)L_\sigma\Big]\\ &+2m\Big[2a^2L_0\Big(\sum_{k=1}^m\beta_kr+\sum_{k=1}^m\tilde{\beta}_k\Big)+L_U\sum_{k=1}^m\beta_k\Big]\Big\}\|x-y\|^2. \end{aligned}$$

First we show that the operator Φ maps $\mathcal{Y}_r$ into itself. Now

$$\begin{aligned} E\|(\Phi x)(t)\|^2 &\leq 9\Big\{E\big\|U(t,0;x)\big[x_0-H(t_1,t_2,\cdots,t_p,x(t_1),x(t_2),\cdots,x(t_p))\\ &-g\big(0,x(0),\cdots,x(\alpha_m(0))\big)\big]\big\|^2+E\big\|g\big(t,x(t),x(\alpha_1(t)),\cdots,x(\alpha_m(t))\big)\big\|^2\\ &+E\Big\|\int_0^tU(t,s;x)A(s,x(s))g\big(s,x(s),x(\alpha_1(s)),\cdots,x(\alpha_m(s))\big)ds\Big\|^2\\ &+E\Big\|\int_0^tU(t,s;x)f\Big(s,x(s),x(\beta_1(s)),\cdots,x(\beta_{n-1}(s)),\int_0^sh\big(s,\eta,x(\beta_n(\eta))\big)d\eta\Big)ds\Big\|^2\\ &+E\Big\|\int_0^tU(t,s;x)\sigma\Big(s,x(s),x(\gamma_1(s)),\cdots,x(\gamma_q(s))\Big)dw(s)\Big\|^2\\ &+E\Big\|\int_0^tU(t,\mu;x)V(\mu,x)d\mu\Big\|^2+E\Big\|\sum_{0<\tau_k<t}U(t,\tau_k;x)I_k(x(\tau_k^-))\Big\|^2\Big\}\\ &\leq 9\Big\{L_U(\|x_0\|^2+\tilde{L}_H)+L_U[2L_2(m+1)(\|x_0\|^2+\tilde{L}_H)+L_3]+L_2(m+1)r\\ &+L_3+a^2L_U\mathcal{G}+2a^2L_U[L_g(m+1)r+\tilde{L}_g]+2a^2L_U[L_f((n+2L_h)r+2\tilde{L}_h)\\ &+\tilde{L}_f]+2aL_UTr(Q)[L_\sigma(q+1)r+\tilde{L}_\sigma]+2mL_U\Big[\sum_{k=1}^m\beta_kr+\sum_{k=1}^m\tilde{\beta}_k\Big]\Big\} \end{aligned}$$

From $(H9)$ we get $E\|(\Phi x)(t)\|^2\leq r$. Hence Φ maps $\mathcal{Y}_r$ into $\mathcal{Y}_r$. Let $x,y\in\mathcal{Y}_r$, then

$$\begin{aligned} E\|(\Phi x)(t)-(\Phi y)(t)\|^2 &\leq 9\Big\{E\Big\|U(t,0;x)\big[x_0-H(t_1,\cdots,t_p,x(t_1),\cdots,x(t_p))\big]\\ &\qquad-U(t,0;y)\big[x_0-H(t_1,\cdots,t_p,y(t_1),\cdots,y(t_p))\big]\Big\|^2\\ &+E\Big\|U(t,0;x)g\big(0,x(0),x(\alpha_1(0))\cdots,x(\alpha_m(0))\big)\\ &\qquad-U(t,0;y)g\big(0,y(0),y(\alpha_1(0)),\cdots,y(\alpha_m(0))\big)\Big\|^2\\ &+E\Big\|g\big(t,x(t),x(\alpha_1(t)),\cdots,x(\alpha_m(t))\big)\\ &\qquad-g\big(t,y(t),y(\alpha_1(t)),\cdots,y(\alpha_m(t))\big)\Big\|^2\\ &+E\Big\|\int_0^t\Big[U(t,s;x)A(s,x(s))g\big(s,x(s),x(\alpha_1(s)),\cdots,x(\alpha_m(s))\big) \end{aligned}$$

RA Journal of Applied Research
||Volume||3||Issue||01||Pages-822-833||Jan-2017|| ISSN (e): 2394-6709
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$$
\begin{aligned}
&-U(t,s;y)A(s,y(s))g\big(s,y(s),y(\alpha_1(s)),\cdots,y(\alpha_m(s))\big)\Big]ds\Big\|^2\\
&+E\Big\|\int_0^t\Big[U(t,\mu;x)V(\mu,x)-U(t,\mu;y)V(\mu,y)\Big]d\mu\Big\|^2\\
&+E\Big\|\int_0^t\Big[U(t,s;x)f\Big(s,x(s),x(\beta_1(s)),\cdots,x(\beta_{n-1}(s)),\int_0^s h\big(s,\eta,x(\beta_n(\eta))\big)d\eta\Big)\\
&-U(t,s;y)f\Big(s,y(s),y(\beta_1(s)),\cdots,y(\beta_{n-1}(s)),\int_0^s h\big(s,\eta,y(\beta_n(\eta))\big)d\eta\Big)\Big]ds\Big\|^2\\
&+E\Big\|\int_0^t\Big[U(t,s;x)\sigma(s,x(s),x(\gamma_1(s)),\cdots,x(\gamma_q(s)))\\
&-U(t,s;y)\sigma(s,y(s),y(\gamma_1(s)),\cdots,y(\gamma_q(s)))\Big]dw(s)\Big\|^2\\
&+E\Big\|\sum_{0<\tau_k<t}\Big[U(t,\tau_k;x)I_k(x(\tau_k^-))-U(t,\tau_k;y)I_k(y(\tau_k^-))\Big]\Big\|^2\Big\}
\end{aligned}
$$

$$
\begin{aligned}
E\|(\Phi x)(t)-(\Phi y)(t)\|^2 &\le 9\big\{(1+16a^2L_UL_W)(N_1+N_2+N_3+N_4+N_5+N_6)\\
&\quad +2a^3L_0\mathcal{G}\big\}\|x-y\|^2\\
&\le \nu\|x-y\|^2.
\end{aligned}
$$

Since $\nu<1$, the mapping Φ is a contraction and hence by Banach fixed point theorem there exists a unique fixed point $x\in\mathcal{Y}_r$ such that $(\Phi x)(t)=x(t)$. This fixed point is then the solution of the system (1.1) and clearly, $x(a)=(\Phi x)(a)=x_1$ which implies that the system (1.1) is controllable on J.

4 Example

Consider the following partial integrodifferential equation of the form

$$
\begin{aligned}
\partial\Big(z(t,y)-\cos z(\sin t,y)\Big) &= \Big(\frac{\partial^3}{\partial y^3}z(t,y)+z(t,y)\frac{\partial}{\partial y}z(t,y)+\mu(t,y)\\
&\quad+\frac{\sin z(t,y)}{(1+t)(1+t^2)}+\int_0^t\frac{z(\sin s,y)}{(1+t)(1+t^2)^2(1+s)^2}ds\Big)\partial t\\
&\quad+e^{-t}(t+1)z(\sin t,y)dw(t),\ t\ne\tau_k,\\
z(t,0) &= z(t,\pi)=0,\ t\in J:=[0,a],\\
z(0,y) &+ \sum_{i=0}^{p}\int_0^\pi k(y,\xi)z(t_i,\xi)d\xi=z_0(y),\ 0\le y\le\pi, \qquad (4.1)\\
\triangle z|_{t=\tau_k} &= I_k(z(y))=(\alpha_k|z(y)|+\tau_k)^{-1},\quad k=1,2,\cdots,m.
\end{aligned}
$$

where p is a positive integer and $0<t_1<t_2<\cdots<t_p<a\le\pi$, $z_0(y)\in H=L^2([0,\pi])$ and $k(y,\xi)\in L^2([0,\pi]\times[0,\pi])$. Here $\alpha_k, k=1,2,\cdots,m$ are constants. For every real s we introduce a Hilbert space $H^s([0,\pi])$ as follows [25]. Let $z\in L^2([0,\pi])$ and set

$$
\|z\|_s=\Big(\int_0^\pi(1+\xi^2)^s|\widehat{z}(\xi)|^2d\xi\Big)^{1/2},
$$

RA Journal of Applied Research
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where $\widehat{z}$ is the Fourier transform of z. The linear space of functions $z \in L^2([0,\pi])$ for which $\|z\|_s$ is finite is a pre-Hilbert space with the inner product

$$(z,y)_s = \Big(\int_0^\pi (1+\xi^2)^s \widehat{z}(\xi)\overline{\widehat{y}(\xi)}d\xi\Big)^{1/2}.$$

The completion of this space with respect to the norm $\|\cdot\|_s$ is a Hilbert space which we denote by $H^s([0,\pi])$. It is clear that $H^0([0,\pi]) = L^2([0,\pi])$.

Take $H = U = K = L^2([0,\pi]) = H^0([0,\pi])$ and $Y = H^s([0,\pi]), s \geq 3$. Define an operator A_0 by $D(A_0) = H^3([0,\pi])$ and $A_0 z = D^3 z$ for $z \in D(A_0)$ where $D = d/dy$. Then A_0 is the infinitesimal generator of a C_0-group of isometries on H. Next we define for every $v \in Y$ an operator $A_1(v)$ by $D(A_1(v)) = H^1([0,\pi])$ and $z \in D(A_1(v))$, $A_1(v)z = vDz$. Then for every $v \in Y$ the operator $A(v) = A_0 + A_1(v)$ is the infinitesimal generator of C_0 semigroup $U(t,0;v)$ on H satisfying $\|U(t,0;v)\| \leq e^{\beta t}$ for every $\beta \geq c_0\|v\|_s$, where c_0 is a constant independent of $v \in Y$. Let $\mathcal{Y}_r$ be the ball of radius $r > 0$ in Y and it is proved that the family of operators $A(v), v \in \mathcal{Y}_r$, satisfies the conditions $(E1)-(E4)$ and $(H1)$ (see [25]).

Put $x(t) = z(t,\cdot)$ and $u(t) = \mu(t,\cdot)$ where $\mu : J \times [0,\pi] \to R$ is continuous,

$$\begin{aligned}
f\Big(t, x(t), x(\beta(t)), \int_0^t h(t,s,x(\beta(s)))ds\Big)(y) &= \frac{\sin z(t,y)}{(1+t)(1+t^2)} + \int_0^t \frac{z(\sin s, y)}{(1+t)(1+t^2)^2(1+s)^2}ds \\
\sigma(t, x(t), x(\gamma(t)))(y) &= e^{-t}(t+1)z(\sin t, y), \\
g(t, x(t), x(\alpha(t)))(y) &= \cos z(\sin t, y), \\
H(t_1,\cdots,t_p, x(t_1),\cdots,x(t_p))(y) &= \sum_{i=0}^p \int_0^\pi k(y,\xi)z(t_i,\xi)d\xi.
\end{aligned}$$

With this choice of $A(v)$, f, g, h, σ, H, I_k, $B = I$, the identity operator and $w(t)$, one dimensional standard wiener process, we see that (4.1) is an abstract formulation of the system (1.1). Further we have

$$\left\|\frac{\sin z(t,y)}{(1+t)(1+t^2)} + \int_0^t \frac{z(\sin s, y)}{(1+t)(1+t^2)^2(1+s)^2}ds\right\| \leq \frac{1}{1+t^2}\|z\|.$$

Assume that the operator $W : L^2(J,U)/KerW \to H$ defined by

$$Wu = \int_0^a U(a,s;x)\mu(s,\cdot)ds$$

has an inverse operator and satisfies condition $(H2)$ for every $x \in \mathcal{Y}_r$.
Further other assumptions $(H3)-(H9)$ are obviously satisfied and it is possible to choose the constants α_k in such a way that the constant $\nu < 1$. Hence, by Theorem 3.1, the system (4.1) is controllable on J.

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