Available online at www.rajournals.in

RA JOURNAL OF APPLIED RESEARCH

ISSN: 2394-6709

DOI:10.31142/rajar/v4i6.04

Volume: 04 Issue: 06 June -2018

International Open Access

ICV- 74.25

Impact Factor :5.985

Page no.- 1731-1738

Estimating Longevity Using Non Parametric and Semi Parametric Survival Functions

Ajayi Moses Adedapo[1], Agbona Anthony Adisa[2], Masopa Adekunle Nurudeen[3]

[1,2,3]Department of Statistics, The Federal Polytechnic, Ede

ARTICLE INFO

Published Online: 05 June 2018

Corresponding Author: **Ajayi Moses Adedapo**

ABSTRACT

The conventional models that are frequently used to summaries the general health of people in different nations are longevity and life expectancy. There are many other different methods of estimating life expectancy and these different methods give widely different answers. When selecting a method for estimating life expectancy, it is important to ensure that the method used is suitable for the data available and for the life history of the respondents. Although there is rarely only one correct method of summarizing demographic information, the problem of life table is that follow cohorts for long periods of time are not common, which prevents cohort analysis, and the critical assumption of a stable-age distribution so difficult to meet. Our derived longevity survival based models form parametric and non-parametric clearly demonstrated that the conventional life table and survival methods are clearly inconsistent and give misleading results. This study utilised University academic retirees data obtained from two premier Universities in Western Nigeria. The estimated mean life expectancy from life table model of Universities academic retirees is 18 years and estimated mean of post retirement years for Universities academic retirees from derived longevity using Kaplan Meier model is 22 years. Utilisation of explanatory variable from derived longevity using Cox proportional model estimated mean of post retirement years for universities academic retirees is 22 years. Based on standard error estimate we can say that life table model is inappropriate for estimating life expectancy.

KEYWORDS: retirees, longevity, life expectancy, life table, Kaplan Meier

I. INTRODUCTION

Estimates of longevity and life expectancy are frequently used in several fields including biostatistics, demography, economics, engineering and sociology (Jerry, 2005). The expressions duration analysis, event-history analysis, failure-time analysis, reliability analysis, and transition analysis refer essentially to the same group of techniques, although the emphases in certain modeling aspects could differ across disciplines (Joseph, 2010).

The concepts of longevity and life expectancy are fairly easy to understand, the longest lived individual and how long individuals live on average, respectively. What is not easily understood is that there are many different methods of estimating life expectancy, and the different methods can give widely different results. When selecting a method for estimating life expectancy, it is important to ensure that the method used is appropriate for the data available and for the life history of the subjects. Although there is rarely only one correct method of brief demographic information, it is often possible to show methods that are clearly incorrect or give misleading results (Robert and Kevin, 2004). The major importance of a life table depends on population's policy for survival and it helps to appreciate the dynamics of populations. In Actuarial Sciences, a life table shows for each age the probability that a person of a certain age will die before his or her next birthday. A number of deductions can be derived; the probability of surviving any particular year of age and remaining life expectancy for people at different ages (Shepard and Robert, 2003).

Life tables are also used extensively in Biology and Epidemiology but put into consideration only one event at a time (Saunders, 2007). The concept is also of importance in product life cycle management (Preston et al 2001). The problem of life table is that, following cohorts for long periods of time are not common, which prevents cohort analysis, and the critical assumption of a stable-age distribution so difficult to meet. An alternative way to alleviate this problem is to use survival analysis in the computation of life expectancy. Survival analysis deals with analysis of time duration to the occurrence of one or more events happen, such as death in biological organisms and failure in mechanical systems (John and Melvin, 2005; and Kate and David, 1997). It tries to examine the proportion of a population which will survive beyond a certain time and, at the rate they will die or fail. The merit of survival analysis is that it takes into consideration the multiple causes of death or failure and increase or decrease probability of survival of particular circumstances or characteristics.

The methodological developments of life expectancy using survival functions that have had the most profound impact are the Kaplan-Meier method for estimating the survival function, the log-rank test for comparing the equality of two or more survival distributions, and the Cox proportional hazards (PH) model for examining the covariate effects on the hazard function (Jiezhi 2009).

In this study, we propose modified life expectancy models infused with survival functions and having underline flavour of non-parametric and semi-parametric survival model. Longevity estimators derived in this study are the extension of conventional models for estimation of life expectancy infused with non-parametric and semi-parametric Survival function with the support of longevity estimation models could be useful in replacing the conventional life expectancy model. This can be used for computing both the expected years of life remaining at a given age and the proportion of a population which will survive beyond a certain time and the rate they will die or fail.

Objective of the study is to derive survival functions based longevity models using non-parametric and semi-parametric survival functions model and apply the derived survival function model based longevity on University academic retiree's data obtained from University of Ibadan (UI) and Obafemi Awolowo University (OAU) in Western Nigeria.

2.0 Survival Function Infused With Life Expectancy Models

This section focuses on derivation of longevity model using survival function and life table functions.

2.1 Life table expectancy model

The expected years of survival in life table model (Anousschka 2012, USAID and Ajayi et al 2014) consecutively is given as

$$e(x)=\frac{T(x)}{l(x)} \tag{1}$$

$$e(x)=\frac{\sum_{Lp}^{Fp} nL(x)}{l(x)} \tag{2}$$

Where $T(x)$ is the total life years lived by the cohort after age x, $l(x)$ is the number of survivors in the life table at exact age x out of an initial population call radix at age 0. $nL(x)$ or $L(x,n)$ is life years lived by the total population between ages x and x+n and where "n" is the age interval, Fp is the first point or first age and Lp is last point or last age in the observation.

Estimating variance of an estimated life expectancy given in Equation 2 (Botman et al, 2000), under an assumption that grouped data in the age intervals are independent across intervals and total deaths (d_i) within interval i, can be modeled as having a binomial distribution with probability q_i but with unknown number of independent trials, Chiang developed Taylor-Linearization methods to estimate $Var\left[\hat{e}(x)\right]$, where a "hat" denotes an estimated quantity:

$$Var\left[\hat{e}(x)\right]=\sum_{i=1}^{n}\hat{p}_i^{\,2}\left[(1-a_i)n_i+\hat{e}_{i+i}\right]^2 Var(\hat{p}_i) \tag{3}$$

Where n_i is the length of interval i and a_i is the average fraction of n_i lived by those who die within the interval assumed to be 0.5 for every interval except the last, p_i is the probability of surviving to age x given survival to age $x+i$.

$$Var(\hat{p}_i)=\left[\frac{\hat{q}_i^2(1-\hat{q}_i)}{d_i}\right] \text{ and } q_i=1-p_i$$

Thus, from (7.2) we have:

$$nL(x)=L(x,n)=0.5n\big(l(x)+l(x+n)\big),$$

$$e(x)=\frac{\sum_{Lp}^{Fp}0.5n\left[l(x)+l(x+n)\right]}{l(x)}$$

Where $\sum_{Lp}^{Fp}$ is the summation from oldest age to the youngest age

$$e(x)=\frac{0.5n\left(\sum_{Lp}^{Fp}\left[l(x)+l(x+n)\right]\right)}{l(x)} \tag{4}$$

But we can express $l(x+n)$ as

$$l(x+n)=P(x)*l(x) \tag{5}$$

Where $p(x)$ is the probability of survival from age x to $x+n$.

If $p(x)=\dfrac{l(x+n)}{l(x)}$, then we have

$$e(x)=0.5n\sum_{Lp}^{Fp}\big(p(x)+1\big)$$

$$e_x=0.5n\sum_{Lp}^{Fp}p(x)+0.5n\sum_{Lp}^{Fp}1 \tag{6}$$

2.3 Kaplan Meier Survival Function Model

The Kaplan Meier estimate of survival function is $\hat{s}(x)=\prod_{\le x}\hat{p}(x)$ and its variance is

$$var(KM)=\hat{s}(t)^2\sum\frac{d_x}{l_x\left(l_x-d_x\right)}.$$

The Kaplan Meier estimate of survival function could be written as:

$$\hat{s}(x)=\hat{p}(x)\times\hat{s}(x-1) \tag{7}$$

Where $p(0)=1$, and $\hat{p}(x)=\dfrac{\hat{s}(x)}{\hat{s}(x-1)}$, then we have

$$\sum_{Lp}^{Fp}\hat{p}(x)=\sum_{Lp}^{Fp}\frac{\hat{s}(x)}{\hat{s}(x-1)} \quad (8)$$

By substituting equation 8 in equation 6, therefore,

$$LG(x)=0.5n\sum_{Lp}^{Fp}\frac{\hat{s}(x)}{\hat{s}(x-1)}+0.5n\sum_{Lp}^{Fp}1$$

$$L\hat{G}(x)=0.5n\left[\sum_{Lp}^{Fp}\left[\hat{s}(x)(\hat{s}(x-1)^{-1})\right]+I_{\geq x}\right] \quad (9)$$

Where $I_{\geq x}$ = summing 1 up to the age x from the highest age.

2.3 Longevity estimation using Kaplan Meier Model ($L\hat{G}(x)_{km}$)

Using this expression the derived Kaplan Meier Model (9) is

$$L\hat{G}(x)_{km}=0.5n\sum_{Lp}^{Fp}\frac{\hat{s}(x)}{\hat{s}(x-1)}+0.5n\sum_{Lp}^{Fp}1$$

$$=(0.5n)\left[\sum_{Lp}^{Fp}\frac{\prod\left[1-\hat{h}(x)\right]}{\prod\left[1-\hat{h}(x-1)\right]}+\sum_{\geq x}^{x}1\right] \quad (10)$$

Equation (9) gives a general form of deduced longevity estimator (LG_x) for selected non parametric, semi-parametric and parametric survival models.

By using the first part of equation (9) that is

$$L\hat{G}(x)_{km}=0.5n\left[\sum_{Lp}^{Fp}\left[\hat{s}(x)(\hat{s}(x-1)^{-1})\right]+I_{\geq x}\right] \quad (11)$$

The variance is of $LG(x)_{KM}$ is

$$Var\left[L\hat{G}(x)_{km}\right]=(0.5n)^2\left[\sum_{Lp}^{Fp}Var\left(\frac{\hat{s}(x)}{\hat{s}(x-1)}\right)\right] \quad (12)$$

Equation (12) involves ratio of estimators and is given as:

$$Var\left(\frac{\hat{s}(x)}{\hat{s}(x-1)}\right)\cong\frac{\mu^2_{\hat{s}(x)}}{\mu^4_{\hat{s}(x-1)}}Var(\hat{s}(x-1))+\frac{1}{\mu^2_{\hat{s}(x)}}Var(\hat{s}(x))$$
$$-2\rho_{\sigma_{\hat{s}(x)}\sigma_{\hat{s}(x-1)}}\mu_{\hat{s}(x)}\mu^{-3}_{\hat{s}(x-1)} \quad (13)$$

Where $\sigma_{\hat{s}(x)}=\sqrt{Var(\hat{s}(x))}$,

$\sigma_{\hat{s}(x-1)}=\sqrt{Var(\hat{s}(x-1))}$ and $\rho=corr\left[\hat{s}(x),\hat{s}(x-1)\right]$

$$Var\left[L\hat{G}(x)_{km}\right]=(0.5n)^2\left[\sum_{Lp}^{Fp}\left(\begin{matrix}\frac{\mu^2_{\hat{s}(x)}}{\mu^4_{\hat{s}(x-1)}}\sigma^2_{\hat{s}(x-1)}+\frac{1}{\mu^2_{\hat{s}(x)}}\sigma^2_{\hat{s}(x)}\\ -2\rho_{\sigma_{\hat{s}(x)}\sigma_{\hat{s}(x-1)}}\mu_{\hat{s}(x)}\mu^{-3}_{\hat{s}(x-1)}\end{matrix}\right)\right] \quad (14)$$

From equation (13) it is required to find Var. ($\hat{s}(x)$) and by mathematical induction we can obtain Var($\hat{s}(x-1)$).

To do this, we have $Var\left(\ln\hat{s}(x)\right)=Var\sum_{\max}^{x}ln\left(1-\hat{h}(x)\right)$ which can be obtained using Delta method (Alex, 2009 and David et. al 2008). But suppose that it is the number of individuals at risk in x_j with d_j as the number of deaths at x_j.

Given that r_j is the total number of individual surviving in the interval $(x_j x_{j+1})$, we can deduce that random number r_j-d_j has a binomial distribution with parameter r_j and $1-\left(\frac{d_j}{r_j}\right)$.

Thus, the conditional variance of r_j-d_j is given by

$$Var\left(r_j-d_j/r_j\right)=r_j\hat{h}_j(1-\hat{h}_j)\text{ ; Therefore}$$

$$Var\left(\hat{h}_j/r_j\right)=Var(1-\hat{h}_j)=\frac{\hat{h}_j(1-\hat{h}_j)}{r_j} \quad (15)$$

By Delta method, we have

$$Var\left[ln\left(\hat{s}(x_j/r_j)\right)\right]=Var\sum_{\max}^{x}ln(1-\hat{h}(x_j/r_j))$$

$$=\sum_{j:x_j<x}^{r}\left(\frac{d}{d\hat{h}(x_j)}\left(ln\left((1-\hat{h}(x_j))\right)\right)\right)^2Var(\hat{h}(x_j/r_j))$$

$$=\sum_{1}^{r}\left(\frac{1}{1-\hat{h}(x_j)}\right)^2\left(\frac{\hat{h}(x_j)(1-\hat{h}(x_j))}{r_j}\right)$$

$$=\sum_{1}^{r}\frac{\hat{h}(x_j)}{r_j(1-\hat{h}(x_j))} \qquad \text{Where j = 1, 2... r}$$

Replacing $h(x_j)$ with $d(x_j)/r_j$

gives $$Var\left[ln\left(\hat{s}(x/r_j)\right)\right]=\sum_{j:x<x}\left[\frac{d_j}{r_j(r_j-d_j)}\right]$$

Finally, if $z=ln\left(\hat{s}(x)\right)$ then $e^z=e^{lns(x)}$, therefore

$$Var(\hat{s}(x))=Var(\hat{e}^z)=\left[\frac{d}{dz}\hat{e}^z\right]^2Var(\hat{z})$$

$$=\left(\hat{s}(x)\right)^2Var(ln\hat{s}(x))$$

$$=(\hat{s}(x))^2\sum_{j:x_{j<x}}\frac{d_j}{r_j(r_j-d_j)} \quad (16)$$

Also

$$Var(\hat{s}(x-1)=(\hat{s}(x-1))^2\sum_{j:x_{j<x-1}}\frac{d_j}{r_j(r_j-d_j)} \quad (17)$$

Let $\sigma_{\hat{s}(x)} = \sqrt{Var(\hat{s}(x)) = (\hat{s}(x))}\left[\sum_{j:x_{j<x}} \frac{d_j}{r_j(r_j-d_j)}\right]^{1/2}$ $\sigma_{\hat{s}(x-1)} = \sqrt{Var(\hat{s}(x-1) = (\hat{s}(x-1)}\left[\sum_{j:x_{j<x-1}} \frac{d_j}{r_j(r_j-d_j)}\right]^{1/2}$.

Therefore the required variance of $LG(x)_{km}$ is

$$Var\left[L\hat{G}(x)_{km}\right] = (0.5n)^2 \left\{ \begin{array}{l} \sum_{Lp}^{Fp}\left[\frac{\mu^2_{\hat{s}(x)}}{\mu^4_{\hat{s}(x-1)}} Var(\hat{s}(x-1)) + \frac{1}{\mu^2_{\hat{s}(x)}} Var(\hat{s}(x))\right] - \\ 2\rho_{s(x)\hat{s}(x-1)}\mu_{\hat{s}(x)}\mu^{-3}_{\hat{s}(x-1)}\left[\sum_{j:x_{j<x}} \frac{d_j}{r_j(r_j-d_j)}\right]^{1/2}\left[\sum_{j:x_{j<x-1}} \frac{d_j}{r_j(r_j-d_j)}\right]^{1/2} \end{array} \right\} \tag{18}$$

The estimator used for the mean is obtained from mathematical result which state that, for a positive continuous random variable, the mean is equal to the area under the survivorship function. From mathematical methods of calculus, this may be represented as the integral of the survivorship function over the range. That is: $\mu = \int_0^\infty S(u)du$. If we restrict the variable to the interval (0,t*), then the mean of the variable in this interval is and $\mu(t^*) = \int_0^{t^*} S(u)du$ (David and Stanley, 1999). The equation defining the estimator based on the observed range of survival time is

$$\mu(x_m) = \sum_{i=1}^{m} \hat{S}\left(x_{(i-1)}\right)\left(x_{(i)} - x_{(i-1)}\right) \tag{19}$$

However, there may be situations where there is considerable uncertainty in measuring the longest censored time, when the estimator based on survival time only is preferred. The Equation (7.18) becomes:

$$\mu(x_n) = \hat{\mu}(x_m) + (1-c_n)\hat{S}\left(x_{(m)}\right)\left(x_{(n)} - x_{(m)}\right) \tag{20}$$

Therefore $\mu_{\hat{S}(x)} = \sum_i^n \hat{S}(x) \times n$

and $\mu_{\hat{S}(x-1)} = \sum_i^n \hat{S}(x-1) \times n$.

2.4 Longevity estimation using Cox Proportional (Cp) Model

The Cox proportional hazard (CPH) model is given by

$$h(x/y) = h_0(x)\exp(\beta_i y_i + ... + \beta_p y_p) = h_0(x)\exp(\beta' y)$$

$h_0(x)$ is the base line hazard function. This is the hazard function for an individual for which all the variables included in the model are zero. Y = (y_1,.........y_q) are the values of the vector of explanatory variables for a particular individual and β' is the vector of regression coefficient. The survival function is given by

$$\hat{s}(x/y) = \hat{s}_0(x)\exp\left(\sum_{j=1}^{p} \beta'_j y_j\right) \tag{21}$$

also

$$\hat{s}((x-1/y)) = \hat{s}_0(x-1)\exp\left(\sum_{j=1}^{p} \beta'_j y_j\right) \tag{22}$$

Since is the base line survival function at age x, then we can extend to point x-1 which in turn gives the longevity model for CP model as

$$L\hat{G}(x)_{cp} = 0.5n\left\{\sum_{Lp}^{Fp}\left[\hat{s}_0(x)\exp\left(\sum_{j=1}^{p}\beta'_j y_j\right)\right]\left[\hat{s}_0(x-1)\exp\left(\sum_{j=1}^{p}\beta'_j y_j\right)\right]^{-1} + I_{\geq x}\right\} \tag{23}$$

$$L\hat{G}(x)_{cp} = 0.5n\left\{\sum_{Lp}^{Fp}\left[\hat{s}_0(x)\right]\left[\hat{s}_0(x-1)^{-1}\right] + I_{\geq x}\right\} \tag{24}$$

Therefore, the estimators used for the variances σ^2_x, σ^2_{x-1}, and means $\mu_{\hat{S}(x)}$, $\mu_{\hat{S}(x-1)}$ are obtained from Equations 16, 17 and 19 respectively.

The variance of Cox proportional longevity model could be derived using Equation (18)

$$Var\left[L\hat{G}(x)_{cp}\right]=(0.5n)^2\left\{\begin{array}{l}\sum_{Lp}^{Fp}\left[\frac{\mu_{\hat{s}_0(x)}^2}{\mu_{\hat{s}_0(x-1)}^4}Var(\hat{s}_0(x-1))+\frac{1}{\mu_{\hat{s}(x)}^2}Var(\hat{s}_0(x))\right]\\ -2\rho_{\hat{s}_0(x)\hat{s}_0(x-1)}\mu_{\hat{s}_0(x)}\mu_{\hat{s}_0(x-1)}^{-3}\left[\sum_{j:x_{j<x}}\frac{d_j}{r_j(r_j-d_j)}\right]^{1/2}\left[\sum_{j:x_{j<x-1}}\frac{d_j}{r_j(r_j-d_j)}\right]^{1/2}\end{array}\right\} \tag{25}$$

The Equation (25) is the same as Equation (14)

3.0 Analysis, Results and Discussion

There are three stages in the sampling procedure adopted for the study. The first stage is purposive. This is based on condition of retirement of the participants (35 years in service and mandatory age 65 years) with pension. The second stage is also purposive. This is the choice of Universities on the basis of large population of retirees and long history of the establishment in Nigeria. The third stage is cluster sampling techniques to select sample size.

The researcher and his trained research assistants personally administered copies of the questionnaire on the sampled retirees at the University of Ibadan. The researcher and research assistants explained all the questions to the respondents in order to enhance appropriate responses from them. From 325 retirees sampled, records indicated that 107 academic retirees have died. Those who are live (218) were given questionnaires in other to confirm information obtained from records and to provide supplementary information. A total of one hundred and ninety-seven (197) respondents were reachable, these represent 90.37% rate of return. Also, the researcher and his trained research assistants personally administered copies of the questionnaire on the cluster sampled retirees at Obafemi Awolowo University (OAU), Ile-Ife. The researcher and research assistants explained all the questions to the respondents in order to enhance appropriate responses from them. From total respondents of 239 retirees in the cluster sampled records show that 175 academic retirees were alive while 64 retirees have died. The respondents that are live were given questionnaires in other to verify information obtained from records and to provide supplementary information. A total of One hundred and sixty-one (161) respondents that were available represent 92% rate of return from OAU academic retirees. Sample data size from both UI and OAU academic retirees is 529.

Demographic Characteristics of UI and OAU are presented in Appendix I. In Appendix I, we observe that the proportion of male respondents from total survey is 74.7%. The result shows that University academic staff members are predominantly male. More so, Appendix I display that 32.34% of the surveyed respondents have died and 67.7% respondents censored. Among the respondents that censored only 34.2% are with their spouse while 33.1% are widowed. From Appendix II, the distribution of retirees by reasons for retirement of UI and OAU academic retirees are presented. Those who retired based on 35 years of service are 54.7% and whereas, those who retired on 65 years age limit is 55.0%, whereas, 45.0% of UI and OAU academic retirees retired on age 65 years. The highest proportion 68.2% of the respondents retired on Cadre of Professor and the lowest proportion 8.1% of the respondents retired on Cadre of lecturer I. It is observed that the respondents that retired at age 60 are 19.5%. In this case, the respondents might have served 35 years in the University system, which is one of the conditions for mandatory retirement. Furthermore, 58.8% of UI and OAU academic retired on age 65 years.

4.0 Application of Conventional Life Table and Derived Longevity Models to UI and OAU Academic Retirees' Data

This section centered on application of classical life table and derived longevity models (non-parametric and semi-parametric) to both UI and OAU academic retirees' data. Appendix II present the results obtained from the classical life table, Kaplan Meier model and derived longevity models with length of service (LOS) as an explanatory variable using "R" package.

Appendix II exhibits that at the beginning of the study, an individual of both UI and OAU academic retirees in his/her first year after retirement is expected to live on average a total of 18.22 years after retirement using conventional life table model. Whereas, an individual could expend a total of about 22 years after his/her of retirement using Kaplan Meier and Cox proportional models for longevity estimation. Furthermore, if we take the example of cohort at age 60 years using usual life table, he/she would expect to live up to about on average 78 years. Whereas, the retirees on the age 60 years could be alive up to 82 years using estimate from Kaplan Meier model. We expect those that are retired at age 60 years to live up to about 82 years on the estimate obtained from Cox proportional model, this utilised (Length of service before retirement) explanatory variable. At the 2nd year of academic retirement, we could see that both UI and OAU academic retirees we live up to 17 years with variance of 0.0976 using the conventional life table model. On the estimate obtained from Kaplan Meier model, both UI and OAU academic retirees will live 21 years with the variance of 0.0078. But UI and OAU academic retirees will live up to 21 years with variance of 0.0077 on the report of estimated longevity using Cox proportional model. The precision of Kaplan Meier model being the best for this cohort group retiring at age 60 years.

In Appendix II, we observe that post retirement years of UI and OAU academic retirees are diminishing linearly. Estimated standard error of life table model is rising from the beginning of retirement year as age of academic retirees' increases. The estimated standard error for longevity estimated using Kaplan Meier and Cox proportional models are reducing from the 1st year of retirement to the 22nd year of retirement for each model. After the 22nd of post retirement, the standard error is not available for KM model. Thus, the precision of our estimates increases as age after retirement increases.

Figure 1 below demonstrated that life expectancy for conventional life table model failed to converge on zero at the end of the 18th year but we discovered that it gives another about 4, 3 to 2 years to live after the 19th, 20th and 21st years of retirement respectively. More so, we discovered that the estimated longevity using Kaplan Meier and Cox proportional models converge to zero at the 22nd year after retirement.

In addition, in Figure 1 below, the estimated longevity using Kaplan Meier and Cox proportional models is much nearer than the traditional life table model. We observed on the graph that the convergence of both estimated longevity using Kaplan Meier and Cox proportional models is obvious on the 22nd and 23rd years. But estimated life expectancy using conventional life table model failed to converge on zero after 18th.

Figure1: Comparisons of estimated life expectancy and derived longevity of both OAU and UI academic retirees at different post retirement age

5.0 Conclusion

The estimated life expectancy from conventional life table model of Universities academic retirees from UI and OAU, we can conclude that mean life expectancy for Universities academic retirees is 18 years as supported a study carried out by Ajayi et all, 2015. Whereas, the estimated mean of post retirement years for Universities academic retirees from derived longevity using Kaplan Meier model is 22 years. More so, with the utilisation of explanatory variable, estimated mean of post retirement years for universities academic retirees is 22 years from derived longevity using Cox proportional model. Based on the result of analyses we can say that life table model is inappropriate for estimating life expectancy because those who censored and failed where treated alike.

More so, life table model is not consistence with its outcome because it gives more years to live than proposed years of life expectancy. We can recommend derived longevity using Kaplan Meier when explanatory variables are not necessary because it is consistent and give preferential treatment to censor and failed respondents simultaneously. Also, derived longevity using Cox proportional model is recommended for estimating lifespan when explanatory variables are present either in the cohort study or follow up study.

6.0 References

1. Ajayi Moses Adedapo, Shangodoyin Dahud Kehinde, Thaga K and Mokgathle Lucky (2014): Estimation of Longevity using Survival Functions, International Journal of Sciences: Basic and Applied Research (IJSBAR) ISSN 2307-4531
2. Anousschka van der Meulen (2012): Life tables and Survival analysis Life tables, Statistics Netherlands.
3. Botman S, Moore T, Moriarity C, and Parsons V. (2000): Design and Estimation for the National Health Interview Survey, 1995-2004, Vital Health Stat 2(130).
4. Jerry A. Hausman (2005): Semi-parametric Methods for Survival and Longitudinal Data, Massachusetts Institute of Technology.
5. Joseph C. Gardiner (2010): Survival Analysis: Overview of Parametric, Nonparametric and Semi parametric approaches and New Developments Division of Biostatistics, Department of Epidemiology, Michigan State University, East Lansing, MI 48824.
6. Robert J. Wiese and Kevin Willis (2004): Calculation of Longevity and Life Expectancy in Captive Elephants TECHNICAL REPORT, Minnesota Zoological Gardens, Apple Valley, Minnesota
7. Shepard, Jon; Robert W. Greene (2003). Sociology and You. Ohio: Glencoe McGraw-Hill. pp. A–22. ISBN 0-07-828576-3.
8. Saunders Comprehensive Veterinary Dictionary, 3 ed. © 2007 Elsevier.
9. Preston, Samuel H.; Patrick Heuveline; Michel Guillot (2001). Demography: measuring and modeling population processes. Blackwell Publishers. ISBN 1-55786-214-1
10. Kate bull and David J. Spiegelhalter (1997): Tutorial in Biostatistics survival analysis in observational studies, Cardiothoracic unit, hospital for sick children, Great Ormond street, London wc1n 3jh, U.K.
11. John P. Klein and Melvin L. Moeschberger (2005): Survival Analysis: Techniques for Censored and Truncated, Spinger.
12. David W. Hosmer and Stanley Lemeshow (1999): Applied Survival Analysis, Regression Modeling of time to Event Data, John Wiley & Sons, INC.
13. Jiezhi Qi (2009): Comparison of Proportional Hazards and Accelerated Failure Time Models, University of Saskatchewan Saskatoon, Saskatchewan
14. USAID,(Measure and Evaluation): Overview of Life Tables and Survival Rates.

References

The heading of the References section must not be numbered. All reference items must be in 8 pt font. Please use Regular and Italic styles to distinguish different fields as shown in the References section. Number the reference items consecutively in square brackets (e.g. [1]).

1. Ding, W. and Marchionini, G. 1997 A Study on Video Browsing Strategies. Technical Report. University of Maryland at College Park.
2. Tavel, P. 2007 Modeling and Simulation Design. AK Peters Ltd.
3. Sannella, M. J. 1994 Constraint Satisfaction and Debugging for Interactive User Interfaces. Doctoral Thesis. UMI Order Number: UMI Order No. GAX95-09398., University of Washington.
4. Brown, L. D., Hua, H., and Gao, C. 2003. A widget framework for augmented interaction in SCAPE.
5. Y.T. Yu, M.F. Lau, "A comparison of MC/DC, MUMCUT and several other coverage criteria for logical decisions", Journal of Systems and Software, 2005, in press.
6. Spector, A. Z. 1989. Achieving application requirements. In Distributed Systems, S. Mullende
7. Forman, G. 2003. An extensive empirical study of feature selection metrics for text classification. J. Mach. Learn. Res. 3 (Mar. 2003), 1289-1305.
8. Fröhlich, B. and Plate, J. 2000. The cubic mouse: a new device for three-dimensional input. In Proceedings of the SIGCHI Conference on Human Factors in Computing Systems.
9. Bowman, M., Debray, S. K., and Peterson, L. L. 1993. Reasoning about naming systems. .

Appendix 1: Demographic Characteristics of cohort of both UI and OAU academic retirees

x	l_x	w_x	d_x	p_x	s_x	e_x	$std.var\ (e)$	$LG_{(x)km}$	$std.var\left(LG_{(x)km}\right)$	$LG_{(x)cp}$	$std.var\left(LG_{(x)cp}\right)$
1	529	2	4	0.99	0.99	18.33	0.0704	21.59	N/A	21.54	N/A
2	523	4	4	0.99	0.98	17.46	0.0976	20.60	0.0078	20.55	0.0077
3	515	7	12	0.98	0.96	16.59	0.1469	19.60	0.0075	19.56	0.0076
4	496	4	6	0.99	0.95	15.97	0.1663	18.61	0.0073	18.57	0.0073
5	486	6	11	0.98	0.93	15.16	0.1939	17.62	0.0070	17.57	0.0071
6	469	16	11	0.98	0.91	14.49	0.2171	16.63	0.0068	16.58	0.0070
7	442	36	12	0.97	0.88	13.81	0.2387	15.64	0.0066	15.59	0.0068
8	394	43	11	0.97	0.86	13.15	0.2562	14.65	0.0064	14.61	0.0066
9	340	40	10	0.97	0.83	12.45	0.2704	13.67	0.0062	13.62	0.0064
10	290	29	9	0.97	0.81	11.72	0.2819	12.68	0.0060	12.63	0.0063
11	252	27	3	0.99	0.80	10.95	0.2854	11.70	0.0058	11.65	0.0061
12	222	25	6	0.97	0.77	10.02	0.2912	10.71	0.0057	10.66	0.0059
13	191	27	7	0.96	0.75	9.15	0.2967	9.72	0.0055	9.67	0.0057
14	157	31	7	0.95	0.71	8.30	0.3017	8.74	0.0053	8.68	0.0056
15	119	10	4	0.96	0.69	7.43	0.3040	7.76	0.0051	7.71	0.0054
16	105	14	9	0.91	0.63	6.50	0.3079	6.78	0.0048	6.73	0.0051
17	82	7	12	0.85	0.54	5.63	0.3121	5.82	0.0045	5.77	0.0048
18	63	16	11	0.80	0.44	4.78	0.3150	4.89	0.0041	4.84	0.0044
19	36	4	4	0.88	0.40	3.91	0.3159	3.98	0.0036	3.94	0.0039
20	28	4	4	0.85	0.34	2.94	0.3164	3.03	0.0029	2.99	0.0032
21	20	2	5	0.74	0.25	1.97	0.3167	2.11	0.0020	2.08	0.0023
22	13	2	7	0.69	0.12	0.99	0.3169	1.23	0.001	1.23	0.0013
23	4	0	4	1.00	0.00	N/A	N/A	0.50	N/A	0.53	0.0009

Appendix II: Estimated life expectancy and derived longevity of both UI and OAU academic retirees with LOS as an explanatory variable

VARIABLES	FREQUENCY	PERCENTAGES
Gender		
Males	395	74.7
Females	134	23.3
Total	529	100.0
Current status		
Censor	358	67.7
Fail	171	32.3
Total	529	100.0
Marital Status		
With spouse	181	51.0
Widowed	175	49.0
Total	356	100
Reasons for ret.		
L. of service	262	49.5
Age Limit	267	50.5
Total	529	100
Level. at Ret.		
Prof	352	66.5
Ass. Prof	43	8.1
SL	95	18.0
L1	39	7.4
Total	529	100.00
Age at Retirement		
60	103	19.5
61	37	7.0
62	29	5.5
63	38	7.2
64	16	3.0
65	306	57.8
Total	529	100.0