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The Impact of a Strong Electromagnetic Wave on the Quantum Hall Effect in Cylindrical Quantum Wires

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ABSTRACT

The impact of strong electromagnetic waves Hall effect is studied theoretically in a Cylindrical Quantum Wire (CQW) with infinitely high potential $V(\vec{r}) = 0$ inside the wire and $V(\vec{r}) = \infty$ elsewhere subjected to a dc electric field $\vec{E}_1 = (0, 0, E_1)$, a magnetic field $\vec{B} = (B, 0, 0)$ and a laser radiation $\vec{E}_0 = \vec{E} \sin \Omega t$. By using the quantum kinetic equation method for electrons interacting with Acoustic Phonon (AP) at low temperatures, we obtain analytical expressions for the conductivity tensor and the Hall Coefficient (HC), which are different from in comparison to those obtained for a rectangular quantum wire (RQW) or two-dimensional (2D) electron systems. Numerical calculations are also applied for GaAs/GaAsAl CQW to show the nonlinear dependence of the HC on the electromagnetic wave (EMW) frequency, and the radius, the length characteristic parameters of CQW. Wave function and energy spectrum in a CQW are dissiminar to those in Quantum Wires (QWs). Therefore, all numerical results are different from those in the case of QWs. The most important result is that the HC reaches saturation as the radius, the length of CQW or the EMW frequency increases.

KEYWORDS: Hall effect, quantum wire, phonon.

1. INTRODUCTION

The Hall effect in bulk semiconductors under the influence of EMWs has been studied in much details [1-5]. In Refs, [1, 2] the Hall effect, where a sample is subjected to a crossed time-dependent electric field and magnetic field. In Refs [3-5], we used the quantum kinetic equation method to study the influence of an intense EMW on the HC in parabolic quantum wells with an in-plane magnetic field [3], in rectangular quantum wells with a perpendicular magnetic field [4], in Doped Semiconductor Superlattices with an In-plane Magnetic Field [5], in Doped Semiconductor Superlattices with a Perpendicular Magnetic Field under the influence of a Laser Radiation [6]. In Refs, [7] show clearly the dependency of the Hall conductivity tensor on the external field. The appearance of the peaks of conductivity satisfies the MPR condition. In a recent work [8], we studied Hall Effect in a Rectangular Quantum Wire (RQW) with infinitely high potential and in the presence of a laser radiation, subjected to a crossed dc electric field and magnetic field in the presence of a strong EMW characterized by electric field. The dependence of the HC under EMW in quantum wires with different directions of external fields still remains open for investigation, especially by analytical and computational methods. In this work, Influence of a Strong Electromagnetic Wave on the HC and Hall Conductivity with In-Plane Magnetic Field in a CQW with confined Electron-AP scattering is calculated by using the quantum kinetic equation for electrons. However, wave function and energy spectrum in a CQW are different from those in a RQW and 2D. Therefore, analytical expressions for the HC in a CQW are obtained, different from those in a RQW [8] and in 2D [3-6]. The obtained results are very different in comparison to Quantum wells and RQW. The most important difference is the magnetophonon resonance (MPR) condition in CQW depending on the indices n, n', l, l' (the quantum numbers of electron) and N, N' (the Landau level) so that quantum theory of the HC and Hall conductivity in CQWs under EMW is newly developed. The work is organized as follows. In Sec. 2, describing a CQW with infinitely high potential. The expressions for Hall conductivity and the HC are presented briefly in Sec. 3. Numerical results and discussion are given in Sec. 4. Conclusions are shown in sec. 5.

2. THE QUANTUM HALL EFFECT IN CYLINDRICAL QUANTUM WIRES WITH INFINITELY HIGH POTENTIAL IN THE PRESENCE OF A LASER RADIATION

Wave function and energy spectrum in a CQWs with infinitely high potential $V(\vec{r})=0$ inside the wire and $V(\vec{r})=\infty$ elsewhere subjected to a dc electric field $\vec{E}_1=(0,0,E_1)$ and magnetic field $\vec{B}=(0,B,0)$ in the presence of a strong EMW characterized by electric field $\vec{E}=(0,0,E_0\sin\Omega t)$.

$$\psi_{n,\ell,\vec{k}}(r,\phi,z)=\begin{cases}0 & r>R\\ \dfrac{1}{\sqrt{V_0}}e^{in\phi}e^{i\vec{k}z}\psi_{n,\ell(r)} & r<R,\end{cases} \tag{1}$$

$$\varepsilon_{n,l}(k)=\frac{\hbar^2k_x^2}{2m^*}+\hbar\omega_c(N_p+\frac{n}{2}+\frac{l}{2}+\frac{1}{2})-\frac{1}{2m^*}\left(\frac{eE_1}{\omega_c}\right)^2. \tag{2}$$

Hamiltonian for Electron–AP interacting system in external field can be written as:

$$H=\sum_{n,l,\vec{k}}\varepsilon_{n,l}(\vec{k}-\frac{e}{c}\vec{A}_{(t)})a^+_{n,l,\vec{k}}a_{n,l,\vec{k}}+\sum_{\vec{q}}\omega_{\vec{q}}b^+_{\vec{q}}b_{\vec{q}}+$$
$$+\sum_{n,l,n',l',\vec{k},\vec{q}}\left|C_{\vec{q}}\right|^2\left|I_{n,l,n',l'}(\vec{q})\right|^2a^+_{n,l,\vec{k}+\vec{q}}a_{n',l',\vec{k}}(b_{\vec{q}}+b^+_{-\vec{q}})+\sum_{\vec{q}}\varphi(\vec{q})a^+_{n,l,\vec{k}+\vec{q}}a_{n',l',\vec{k}} \tag{3}$$

Where $a^+_{n,l,\vec{k}}$ and $a_{n,l,\vec{k}}$ ($b^+_{\vec{q}}$ and $b_{\vec{q}}$) are the creation and annihilation operators of electron (OP); $\vec{k}=(0,0,k_z)$ is the electron wave momentum (along the wire's axis: z axis); $\vec{q}$ is the phonon wave vector; $\omega_{\vec{q}}$ are AP frequency; $C_{n,l,n',l'}(\vec{q})=C_{\vec{q}}I_{n,l,n',l'}(\vec{q})$ is the Electron – AP interaction coefficient; $\left|C_{\vec{q}}\right|^2=\frac{\hbar q\xi^2}{2\rho\upsilon_s V}$ ($\vec{q}$ is the phonon wave vector ; υ_s,ξ,ρ,V are the sound velocity, the acoustic deformation potential, the mass density and the normalization volume of specimen, respectively). $\vec{A}(t)=\frac{1}{\Omega}E_0\cos(\Omega t)$ is the potential vector, depending on the external field. $I_{n,l,n',l'}(\vec{q})$ is the electron form factor different from that in RQW and in quantum wells.

$$I_{n,\ell,n'\ell'}(q_\perp)=\frac{2}{R^2}\int_0^R J_{|n-n'|}(q_\perp R)\psi^*_{n',\ell'}(r)\psi_{n,\ell}(q_\perp R)rdr. \tag{4}$$

In which $\psi_{n,\ell}(r)$ is radial wave function:

$$\psi_{n,\ell}(r)=\frac{1}{J_{(n+1)}(A_{n,l})}J_n(A_{n,l}\frac{r}{R}). \tag{5}$$

Where the radial wave function containing $A_{n,l}$ is the root of the Bessel function ($J_n(x)$).

$\varphi(\vec{q})$ is the potential undirected: $\varphi(\vec{q})=(2\pi i)^3(e\vec{E}+\omega_c[\vec{q},\vec{h}])\frac{\partial}{\partial\vec{q}}\delta(\vec{q})$. (6)

Here, $\omega_c=eB/m$ is the cyclotron frequency, R is the radius of wire.

From Hamiltonian (3) for Electron–AP interacting system in a CQW with Infinitely High Potential and the procedures as in the previous work [8], quantum kinetic equations are obtained. After several operator calculations the similar way as in [8] and performing the analytical calculation for the total current density, analytical the expressions for the components σ_{zz} and σ_{zx} of the Hall conductivity are obtained. Then the expression for the HC in CQW for case of Electron – AP scattering can be written as:

$$R_H = \frac{1}{B}\frac{\frac{\omega_c\tau}{1+\omega_c^2\tau^2}\left[\frac{ea}{\omega_c}+\frac{b}{m}\frac{\tau^2}{1+\omega_c^2\tau^2}\right]}{\left\{\frac{-\omega_c\tau}{1+\omega_c^2\tau^2}\left[\frac{ea}{\omega_c}+\frac{b}{m}\frac{\tau^2}{1+\omega_c^2\tau^2}\right]\right\}^2+\left\{\frac{\tau}{1+\omega_c^2\tau^2}\left[ea+\frac{b}{m}\frac{\tau}{1+\omega_c^2\tau^2}(1-\omega_c^2\tau^2)\right]\right\}^2}. \quad (7)$$

Where δ_{ij} is the Kronecker delta; ε_{ijk} being the antisymmetrical Levi – Civita tensor; the Latin h_k, h_i, h_j stand for the components x, y, z of the Cartesian coordinate system;

Where: $a = \frac{L}{2\pi}\frac{e\beta\hbar}{m^2}\frac{\tau_o}{1+\omega_c^2\tau_o^2}\exp\left\{\beta\left[\varepsilon_F - \hbar\omega_c(N+\frac{n}{2}+\frac{l}{2}+\frac{1}{2})+\frac{e^2E_1^2}{2m\omega_c^2}\right]\right\}\left(\frac{2m}{\beta\hbar^2}\right)^{3/2}\frac{\sqrt{\pi}}{2}$,

$$b = \frac{2\pi e}{m}\frac{k_BT\xi^2}{2\rho\upsilon_s^2V}I\sum_{\gamma,\gamma'}(S_1+2S_2+2S_3+2S_4+S_5), c = \exp\left\{\beta\left[\varepsilon_F - \omega_c\left(N+\frac{n}{2}+\frac{l}{2}+\frac{1}{2}\right)+\frac{1}{2m}\left(\frac{eE_1}{\omega_c}\right)^2\right]\right\},$$

$$S_1 = \frac{\beta LmV}{16\sqrt{2}\pi^4}e^{-\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma)/4}\left[(\varepsilon_{\gamma'}-\varepsilon_\gamma)K_o\left(\frac{\varepsilon_{\gamma'}-\varepsilon_\gamma}{2}\right)+\frac{1}{4m^2(\varepsilon_{\gamma'}-\varepsilon_\gamma)}K_1\left(\frac{\varepsilon_{\gamma'}-\varepsilon_\gamma}{2}\right)\right]c,$$

$$S_2 = \frac{\beta Lm^3VE_o(\varepsilon_{\gamma'}-\varepsilon_\gamma)}{16\sqrt{2}\pi^2}e^{-\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma)/4}\left[\sqrt{2}K_1\left(\frac{\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma)}{\sqrt{2}}\right)+\frac{1}{\sqrt{2}}K_2\left(\frac{\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma)}{\sqrt{2}}\right)\right]c,$$

$$S_3 = \frac{\beta Lm^3VE_o(\varepsilon_{\gamma'}-\varepsilon_\gamma+\Omega)}{16\sqrt{2}\pi^2}e^{-\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma+\Omega)/4}\left[\sqrt{2}K_1\left(\frac{\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma+\Omega)}{\sqrt{2}}\right)+\frac{1}{\sqrt{2}}K_2\left(\frac{\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma+\Omega)}{\sqrt{2}}\right)\right]c,$$

$$S_4 = \frac{\beta Lm^3VE_o(\varepsilon_{\gamma'}-\varepsilon_\gamma-\Omega)}{16\sqrt{2}\pi^2}e^{-\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma+\Omega)/4}\left[\sqrt{2}K_1\left(\frac{\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma-\Omega)}{\sqrt{2}}\right)+\frac{1}{\sqrt{2}}K_2\left(\frac{\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma-\Omega)}{\sqrt{2}}\right)\right]c, \quad I = \int_{-\infty}^{+\infty}\left|I_{\gamma,\gamma'}(\vec{q})\right|^2 d\vec{q}$$

$$S_5 = \frac{\beta Lm^2V}{32\sqrt{2}\pi^4}e^{-\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma)/4}\left[(\varepsilon_{\gamma'}-\varepsilon_\gamma)K_o\left(\frac{\varepsilon_{\gamma'}-\varepsilon_\gamma}{\sqrt{2}}\right)+\frac{1}{2m}K_1\left(\frac{\varepsilon_{\gamma'}-\varepsilon_\gamma}{\sqrt{2}}\right)\right]c, \quad \beta = 1/(k_BT),$$

ε_F is the Fermi level. τ is the momentum relaxation time; $I_{\gamma,\gamma'}(\vec{q})$ is The electron form factor; k_B is Boltzmann constant; T is temperature. $K_i(x)$ are modified Bessel functions.

Equation (7) shows the dependency of the HC on the size of wire, including the EMW. It is obtained for arbitrary values of the indices n, n', l, l', N, N' . Furthermore, it is seem that the change wires have modified the wave function and energy spectrum of electrons and, consequently, the obtained results are now very different from our previous results in quantum wells, bulk semiconductors and rectangular quantum wire. The obtained results are very different in comparison to Quantum wells, RQW and 1DEG without EMW.

3. NUMERICAL RESULTS AND DISCUSTIONS

In this section, we present detailed numerical calculations dependence of the HC on the length of quantum wires of the system when the temperature changes with parameters [8]:

$$\varepsilon = 12.5, m = 6.006\times10^{-24}, \varepsilon_F = 50meV, \hbar\omega_0 = 36.25meV, \Omega = 3\times10^{13}s^{-1}, k_B = 1.38\times10^{-23}kg/m^3$$

$$N - N' = 1, n = 1, n' = 0\div1, l = 1, l' = 0\div1, \tau = 10^{-12}s, \rho = 5320kgm^{-3}, q = 2\times10^5m^{-1}, \upsilon_s = 5220m/s, \xi = 2.2\times10^{-18}J$$

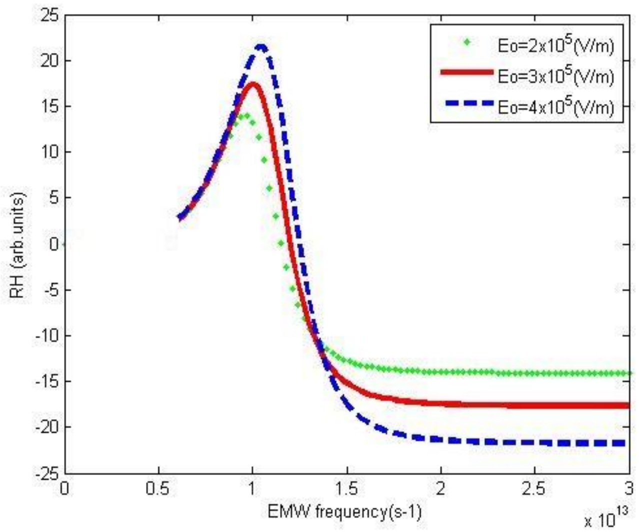

Figure 1: The dependence of the HC with frequency EMW at different values of EMW

The dependence of the HC on the EMW frequency is shown in Fig 1 at different values of EMW. The HC can be seen to oscillate slightly with the change of EMW frequency in the small region. When the frequency is increased continuosly, the HC reaches saturation. This behavior is different to case of the in-plane magnetic field with optical phonon interaction in doped semiconductor superlattices. In doped semiconductor superlattices, the HC can be seen to increase strong with increasing EMW frequency for the region of small value and reaches saturation as the EMW frequency continues to increase. In rectangular quantum wire, as the frequency rises, the HC increase before reaching a peak at certain frequency. Then, it falls sharply and then fell sharply. And if the EMW frequency keep increasing, the HC will remain constant. At these values from different the of EMW, different shape figures. There is no difference between the maximum value of the HC. It is seem that besides the main resonant peaks, as in the case of the absence of the EMW. In this case the HC has both negative and positive values. As frequency EMW increases, the HC is positive, reaches the maximum value and then decreases suddebly to a minimum with a negative value. That's the difference for HC in 2D (quantum wells, Doped semiconductor superlattices [6, 7]) systems only positive values.

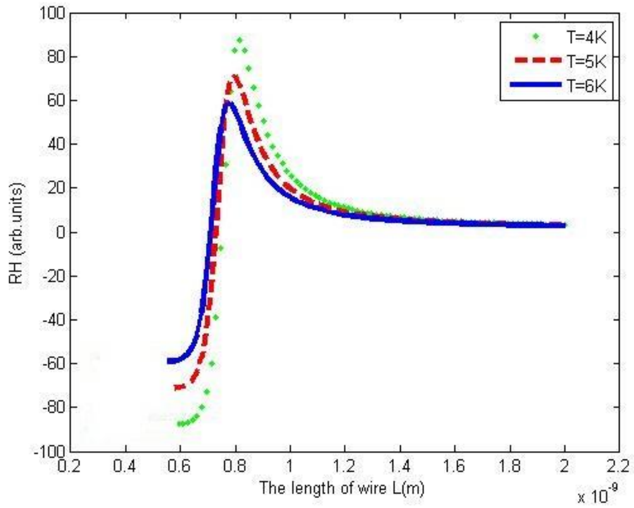

Figure 2: The dependence of the HC on L(m) of CQW

Figure 2 is also seen that the value of the length at which the saturation reaches depends on the characteristic of quantum wires. The length L (m) keeps rising at this time the properties of the wire are similar to those of bulk semiconductor. It means that HC is no longer dependent on the length quantum wires at all different temperatures, which is the characteristic of bulk semiconductor. The HC reaches saturation. This behavior is similar to the case of the dependence of the HC on the length of RQW. However in RQW, the HC depends on wire size of RQW Lx and Ly at different values of temperatures [8]. This is the difference between CQW and RQW.

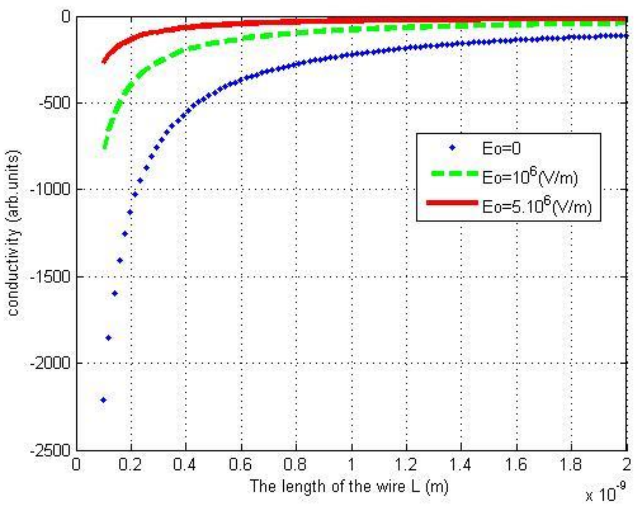

Figure 3: The dependence of the Hall Conductivity on the length of the CQWs L(m) at different values of EMW

We can see from the figure 3 that the dependence of the Hall Conductivity on the length of quantum wires is nonlinear. The Hall Conductivity increases rapidly as the length of quantum wire increase within the domain of small values. If the length continues to increase, the Hall Conductivity will change before remaining constant. With the different values of EMW, the shapes of graphs are virtually unchanged. Therefore, the Hall Conductivity can be considered independing upon EMW.

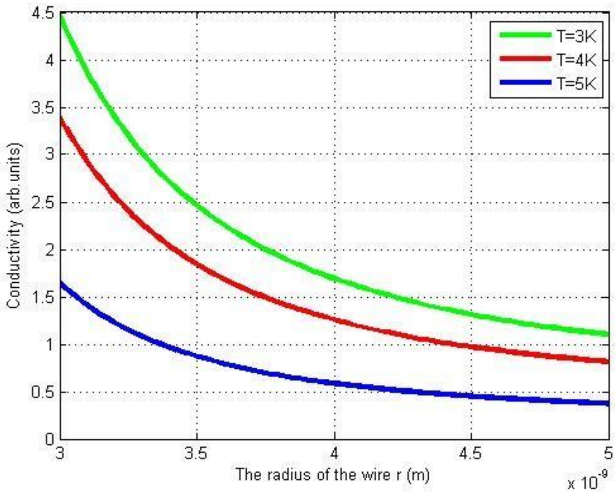

Figure 4: The dependence of the Conductivity on the radius of the wire r(m)of the CQWs

Figure 4 shows the dependence of the conductivity on the radius of the wire r (m) at different temperature is nonlinear. The conductivity decreases rapidly as the radius of quantum wire increase within the domain of small values. If the radius of the wire continues to increase, the conductivity will change before remaining constant. With the different temperatures, the shapes of graphs are virtually unchanged. The larger the wire radius, quantum wires become bulk semiconductors. Meanwhile the conductivity no longer depends on the radius of quantum wire. This is true in bulk semiconductors.

4. CONCLUSION

In this work, the influence of laser radiation on the Hall effect in a cylindrical quantum wires with infinitely High potential subjected to a crossed dc electric and magnetic fields has been studied. The electron – acoustic phonon interaction is taken into account. The expressions of Hall conductivity as well as HC are obtained. The analytical results are numerically evaluated and plotted for a specific quantum wires GaAs/AlGaAs to show clearly the dependence of Hall conductivity on the length and the radius of wire, when the length and the radius of wire to increase, the Hall conductivity will change. When the size of the wire is large, the Hall conductivity has saturation value. The dependence of HC with the frequency of EMW and the length of quantum wires are plotted. The dependence of the HC with frequency electromagnetic: Initially, as the frequency rises, the HC increase before reaching a peak at certain frequency. Then, it falls sharply. And if the EMW frequency keeps increasing, the HC will remain constant. At these values from different EMW, different shape figures, There is no difference between the maximum values of the Hall coefficients.

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