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# Influence of a Temperatures on the Quantum Hall Effect in Cylindrical Quantum Wires with Asymmetrical Parabolic Potential

**Hoang Dinh Trien**

Faculty of Physics, University of Science and Education, The University of Danang 459-Ton Duc Thang, Lien Chieu, Danang

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**Corresponding Author:**
Hoang Dinh Trien

**ABSTRACT**

The Hall effect is studied theoretically in a Cylindrical Quantum Wire (CQW) with asymmetrical parabolic potential $V(x,y) = \frac{m}{2}(\Omega_x^2 x^2 + \Omega_y^2 y^2)$ to a dc electric field $\vec{E}_1 = (0,0,E_1)$, a magnetic field $\vec{B} = (B,0,0)$ and a laser radiation $\vec{E}_0 = \vec{E}\sin\Omega t$ (where $E_0$ and $\Omega$ are the amplitude and the frequency of the laser radiation, respectively). By using the quantum kinetic equation method for electrons interacting with Acoustic Phonon (AP), we obtain analytical expressions for the conductivity tensor and the Hall Coefficient (HC). Numerical calculations are also applied for GaAs/GaAsAl CQW to show the nonlinear dependence of the HC on the temperatures of system.

**KEYWORDS:** Hall effect, Cylindrical quantum wire, asymmetrical parabolic potential.

## 1. INTRODUCTION

It is well known that in one dimensional systems, the motion of electrons is restricted in two dimensions, so that they can flow freely in one dimension. The confinement of electron in these systems has changed the electron mobility remarkably. This has resulted in a number of new phenomena, which concern a reduction of sample dimensions. These effects differ from those in bulk semiconductors. The Hall effect in bulk semiconductors under the influence of EMWs has been studied in much details [1-5]. In Refs, [1, 2] the Hall effect, where a sample is subjected to a crossed time-dependent electric field and magnetic field. In Refs [3-5], by using the quantum kinetic equation method to study the influence of an intense EMW on the HC in parabolic quantum wells with an in-plane magnetic field [3], in rectangular quantum wells with a perpendicular magnetic field [4], in Doped Semiconductor Superlattices with an In-plane Magnetic Field [5], in Doped Semiconductor Superlattices with a Perpendicular Magnetic Field under the influence of a Laser Radiation [6]. In Refs, [7] show clearly the dependency of the Hall conductivity tensor on the external field. The appearance of the peaks of conductivity satisfies the MPR condition. In a recent work [8], the Hall Effect in a Rectangular Quantum Wire (RQW) with infinite high potential and in the presence of a laser radiation, subjected to a crossed dc electric field and magnetic field in the presence of a strong EMW characterized by electric field has also been studied. But in Cylindrical Quantum Wire with asymmetrical parabolic potential, Hall Effect is still open for studying. In this paper, we use the quantum kinetic quation for electrons to theoretically study the dependence of the Hall coefficient in CQW on the temperature T of the system. The problem is considered in electron-acoustic phonon scattering. Numerical calculations are carried out with a specific GaAs/GaAsAl quantum wires to show the dependence of the Hall coefficient in cylindrical quantum wire with asymmetrical parabolic potential on the temperature T of the system.

## 2. THE QUANTUM HALL EFFECT IN CYLINDRICAL QUANTUM WIRES WITH ASYMMETRICAL PARABOLIC POTENTIAL

In our model, we consider a Cylindrical Quantum Wire (CQW), the carriers (electron gas) are assumed to be confined by an asymmetrical parabolic potential $V(x,y) = \frac{m}{2}(\Omega_x^2 x^2 + \Omega_y^2 y^2)$, where $\Omega_x$ and $\Omega_y$ are the effective frequencies of the potential pit in the two quantized directions x and y.

Wave function and energy spectrum in a CQW with asymmetrical parabolic potential to a dc electric field $\vec{E}_1 = (0,0,E_1)$ and magnetic field $\vec{B} = (0,B,0)$ in the presence of a strong EMW characterized by electric field $\vec{E} = (0,0,E_0\sin\Omega t)$.

$$\psi_{n,\ell,\vec{p}}(x,y,z)=\frac{e^{ipz}}{\sqrt{L}}\frac{1}{2^n n! l_x\sqrt{\pi}}e^{-x^2/2l_x^2}H_n(\frac{x}{l_x})\frac{1}{2^\ell \ell! l_y\sqrt{\pi}}e^{-y^2/2l_y^2}H_n(\frac{y}{l_y}), \qquad (1)$$

$$\varepsilon^H_{n,\ell}(\vec{p})=\frac{\vec{p}^2}{M}+\omega_1(N+n+1/2)+\omega_2(N+\ell+1/2), \qquad (2)$$

where $\omega_c=eB/mc$ is cyclotron frequency, $\omega_1^2=\Omega_x^2+\omega_c^2, \omega_2^2=\Omega_y^2+\omega_c^2$, $l_x=\sqrt{1/4m\Omega_x}$ , $l_y=\sqrt{1/4m\Omega_y}$ , và $M=m[1+(\omega_c/\Omega_y)^2+(\omega_c/\Omega_x)^2]$, $H_n(x)$ is Hermite polynomial.

Hamiltonian for Electron–phonon interacting system in external field can be written as:

$$H=\sum_{n,l,\vec{k}}\varepsilon_{n,l}(\vec{k}-\frac{e}{c}\vec{A}_{(t)})a^+_{n,l,\vec{k}}a_{n,l,\vec{k}}+\sum_{\vec{q}}\omega_{\vec{q}}b^+_{\vec{q}}b_{\vec{q}}+$$
$$+\sum_{n,l,n',l',\vec{k},\vec{q}}\left|C_{\vec{q}}\right|^2\left|I_{n,l,n',l'}(\vec{q})\right|^2 a^+_{n,l,\vec{k}+\vec{q}}a_{n',l',\vec{k}}(b_{\vec{q}}+b^+_{-\vec{q}})+\sum_{\vec{q}}\varphi(\vec{q})a^+_{n,l,\vec{k}+\vec{q}}a_{n',l',\vec{k}} \qquad (3)$$

Where $a^+_{n,l,\vec{k}}$ and $a_{n,l,\vec{k}}$ ($b^+_{\vec{q}}$ and $b_{\vec{q}}$) are the creation and annihilation operators of electron (OP); $\vec{k}=(0,0,k_z)$ is the electron wave momentum (along the wire's axis: $z$ axis); $\vec{q}$ is the phonon wave vector; $\omega_{\vec{q}}$ are AP frequency; $C_{n,l,n',l'}(\vec{q})=C_{\vec{q}}I_{n,l,n',l'}(\vec{q})$ is the Electron – phonon interaction coefficient; $\left|C_{\vec{q}}\right|^2=\frac{\hbar q\xi^2}{2\rho\upsilon_s V}$ ($\vec{q}$) is the phonon wave vector ; $\upsilon_s,\xi,\rho,V$ are the sound velocity, the acoustic deformation potential, the mass density and the normalization volume of specimen, respectively). $\vec{A}(t)=\frac{1}{\Omega}E_0\cos(\Omega t)$ is the potential vector, depending on the external field. $I_{n,l,n',l'}(\vec{q})$ is the electron form factor and can be written as

$$I_{n,\ell,n'\ell'}(q_\perp)=\frac{2}{R^2}\int_0^R\psi_{n,\ell}(r)e^{iqr}\psi^*_{n',\ell'}(r)rdr. \qquad (4)$$

$\varphi(\vec{q})$ is the potential undirected: $\varphi(\vec{q})=(2\pi i)^3(e\vec{E}+\omega_c[\vec{q},\vec{h}])\frac{\partial}{\partial\vec{q}}\delta(\vec{q}).$, (5)

R is the radius of wire.

In order to establish analytical expressions for the nonlinear absorption coefficient of a strong EMW by confined electrons in RQW, we use the quantum kinetic equation for particle number operator of electron $n_{\gamma,\vec{p}}(t)=\langle a^+_{\gamma,\vec{p}}a_{\gamma,\vec{p}}\rangle_t$

$$i\frac{\partial n_{\gamma,\vec{p}}(t)}{\partial t}=\langle[a^+_{\gamma,\vec{p}}a_{\gamma,\vec{p}},H]\rangle_t \qquad (6)$$

From Eq.(6), using Hamiltonian in Eq.(3) and realizing calculations, we obtain quantum kinetic equation for confined electrons in CQW:

$$\frac{\sum_{\gamma,\vec{k}}\frac{e}{m}\vec{k}n_{\gamma,\vec{k}}\delta(\varepsilon-\varepsilon_{\gamma,\vec{k}})}{\tau}+\omega_c\left[\vec{h},\sum_{\gamma,\vec{k}}\frac{e}{m}\vec{k}n_{\gamma,\vec{k}}\delta(\varepsilon-\varepsilon_{\gamma,\vec{k}})\right]=$$
$$=-\frac{e}{m}\sum_{\gamma,\vec{k}}\vec{k}\left(\vec{F}\frac{\partial n_{\gamma,\vec{k}}}{\partial\vec{k}}\right)\delta(\varepsilon-\varepsilon_{\gamma,\vec{k}})+\frac{2\pi e}{m}\sum_{\gamma,\gamma',\vec{q},\vec{k}}\left|C_{\vec{q}}\right|^2\left|I_{\gamma,\gamma'}(\vec{q})\right|^2N_{\vec{q}}\vec{k}\times$$
$$\times\left\{[\bar{n}_{\gamma',\vec{q}+\vec{k}}-\bar{n}_{\gamma,\vec{k}}]\left[\left(1-\frac{\lambda^2}{2\Omega^2}\right)\delta(\varepsilon_{\gamma',\vec{k}+\vec{q}}-\varepsilon_{\gamma,\vec{k}}-\omega_o)+\frac{\lambda^2}{4\Omega^2}\delta(\varepsilon_{\gamma',\vec{k}+\vec{q}}-\varepsilon_{\gamma,\vec{k}}-\omega_o+\Omega)+\right.\right.$$
$$\left.+\frac{\lambda^2}{4\Omega^2}\delta(\varepsilon_{\gamma',\vec{k}+\vec{q}}-\varepsilon_{\gamma,\vec{k}}-\omega_o-\Omega)\right]+[\bar{n}_{\gamma',\vec{k}-\vec{q}}-\bar{n}_{\gamma,\vec{k}}]\left[\left(1-\frac{\lambda^2}{2\Omega^2}\right)\delta(\varepsilon_{\gamma',\vec{k}-\vec{q}}-\varepsilon_{\gamma,\vec{k}}+\omega_o)+\right.$$
$$\left.\left.+\frac{\lambda^2}{4\Omega^2}\delta(\varepsilon_{\gamma',\vec{k}-\vec{q}}-\varepsilon_{\gamma.\vec{k}}+\omega_o-\Omega)+\frac{\lambda^2}{4\Omega^2}\delta(\varepsilon_{\gamma',\vec{k}-\vec{q}}-\varepsilon_{\gamma,\vec{k}}+\omega_o+\Omega)\right]\right\}\delta(\varepsilon-\varepsilon_{\gamma,\vec{k}}) \qquad (7)$$

Where $\gamma$ and $\gamma'$ are quantum number combination $(n,l,N)$ and $(n',l',N')$.

After several operator calculations the similar way as in [8] and performing the analytical calculation for the total current density, analytical the expressions for the components $\sigma_{zz}$ and $\sigma_{zx}$ of the Hall conductivity are obtained. Then the expression for the HC in CQW for case of Electron – acoustic phonon scattering can be written as:

$$R_H = \frac{1}{B}\frac{\frac{\omega_C\tau}{1+\omega_C^2\tau^2}\left[\frac{ea}{\omega_C}+\frac{b}{m}\frac{\tau^2}{1+\omega_C^2\tau^2}\right]}{\left\{\frac{-\omega_C\tau}{1+\omega_C^2\tau^2}\left[\frac{ea}{\omega_C}+\frac{b}{m}\frac{\tau^2}{1+\omega_C^2\tau^2}\right]\right\}^2+\left\{\frac{\tau}{1+\omega_C^2\tau^2}\left[ea+\frac{b}{m}\frac{\tau}{1+\omega_C^2\tau^2}(1-\omega_C^2\tau^2)\right]\right\}^2} \quad (8)$$

Where $\delta_{ij}$ is the Kronecker delta; $\varepsilon_{ijk}$ being the antisymmetrical Levi – Civita tensor; the Latin $h_k, h_i, h_j$ stand for the components x, y, z of the Cartesian coordinate system;

Where:

,

$$a = \frac{L}{2\pi}\frac{e\beta\hbar}{m_2}\frac{\tau_o}{1+\omega_c^2\tau_0^2}\exp\left\{\beta\left[\varepsilon_F-\omega_1\left(N+n+\frac{1}{2}\right)-\omega_2\left(N+\ell+\frac{1}{2}\right)+\frac{e^2E_1^2}{2m\omega_c^2}\right]\right\}\left(\frac{2m}{\beta\hbar^2}\right)^{\frac{3}{2}}\frac{\sqrt{\pi}}{2}$$

$$b = \frac{2\pi e}{m}\frac{k_BT\xi^2}{2\rho\upsilon_s^2V}I\sum_{\gamma,\gamma'}(S_1+2S_2+2S_3+2S_4+S_5),$$

$$c = exp\left\{\beta\left[\varepsilon_F-\omega_1\left(N+n+\frac{1}{2}\right)-\omega_2\left(N+l+\frac{1}{2}\right)+\frac{1}{2m}\left(\frac{eE_1}{\omega_c}\right)^2\right]\right\},$$

$$S_1 = \frac{\beta LmV}{16\sqrt{2}\pi^4}e^{-\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma)/4}\left[\left(\varepsilon_{\gamma'}-\varepsilon_\gamma\right)K_o\left(\frac{\varepsilon_{\gamma'}-\varepsilon_\gamma}{2}\right)+\frac{1}{4m^2(\varepsilon_{\gamma'}-\varepsilon_\gamma)}K_1\left(\frac{\varepsilon_{\gamma'}-\varepsilon_\gamma}{2}\right)\right]c,$$

$$S_2 = \frac{\beta Lm^3VE_o(\varepsilon_{\gamma'}-\varepsilon_\gamma)}{16\sqrt{2}\pi^2}e^{-\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma)/4}\left[\sqrt{2}K_1\left(\frac{\beta\left(\varepsilon_{\gamma'}-\varepsilon_\gamma\right)}{\sqrt{2}}\right)+\frac{1}{\sqrt{2}}K_2\left(\frac{\beta\left(\varepsilon_{\gamma'}-\varepsilon_\gamma\right)}{\sqrt{2}}\right)\right]c,$$

$$S_3 = \frac{\beta Lm^3VE_o(\varepsilon_{\gamma'}-\varepsilon_\gamma+\Omega)}{16\sqrt{2}\pi^2}e^{-\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma+\Omega)/4}\left[\sqrt{2}K_1\left(\frac{\beta\left(\varepsilon_{\gamma'}-\varepsilon_\gamma+\Omega\right)}{\sqrt{2}}\right)+\frac{1}{\sqrt{2}}K_2\left(\frac{\beta\left(\varepsilon_{\gamma'}-\varepsilon_\gamma+\Omega\right)}{\sqrt{2}}\right)\right]c,$$

$$S_4 = \frac{\beta Lm^3VE_o(\varepsilon_{\gamma'}-\varepsilon_\gamma-\Omega)}{16\sqrt{2}\pi^2}e^{-\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma+\Omega)/4}\left[\sqrt{2}K_1\left(\frac{\beta\left(\varepsilon_{\gamma'}-\varepsilon_\gamma-\Omega\right)}{\sqrt{2}}\right)+\frac{1}{\sqrt{2}}K_2\left(\frac{\beta\left(\varepsilon_{\gamma'}-\varepsilon_\gamma-\Omega\right)}{\sqrt{2}}\right)\right]c,$$

$$S_5 = \frac{\beta Lm^2V}{32\sqrt{2}\pi^4}e^{-\beta(\varepsilon_{\gamma'}-\varepsilon_\gamma)/4}\left[\left(\varepsilon_{\gamma'}-\varepsilon_\gamma\right)K_o\left(\frac{\varepsilon_{\gamma'}-\varepsilon_\gamma}{\sqrt{2}}\right)+\frac{1}{2m}K_1\left(\frac{\varepsilon_{\gamma'}-\varepsilon_\gamma}{\sqrt{2}}\right)\right]c,$$

$\beta = 1/(k_BT)$, $\varepsilon_F$ is the Fermi level. $\tau$ is the momentum relaxation time; $I_{\gamma,\gamma'}(\vec{q})$ is The electron form factor; $k_B$ is Boltzmann constant; T is temperature. $K_i(x)$ are modified Bessel functions.

Equation (8) shows the dependence of the HC on the size of wire, the cyclotron frequency of magnetic field, frequency $\Omega$ of the external strong electromagnetic wave and many other parameters of the system including temperature.

## 3. NUMERICAL RESULTS AND DISCUSTIONS

In order to clarify the dependence of the Hall coefficient in cylindrical quantum wire with asymmetrical parabolic potential on the temperature T, in this section, we numerically calculate the Hall coefficient for a $GaAs/GaAsAl$ CQW. The parameters used in the numerical calculations [8] are $\xi = 13.5\,eV$, $\rho = 5.32\ gcm^{-3}$, $\upsilon_s = 5378\,ms^{-1}$, $\varepsilon_0$=12.5, $\chi_\infty = 10.9$, $\chi_0 = 13.1$, $m = 0.066m_0$, $m_0$ being the mass of free electron, $\hbar\omega = 36.25\,meV$, $k_b = 1.3807\times10^{-23}\ j/K$, $n_0 = 10^{23}\ m^{-3}$, $e = 1.60219\times10^{-19}\ C$, $\hbar = 1.05459\times10^{-34}\ j.s$.

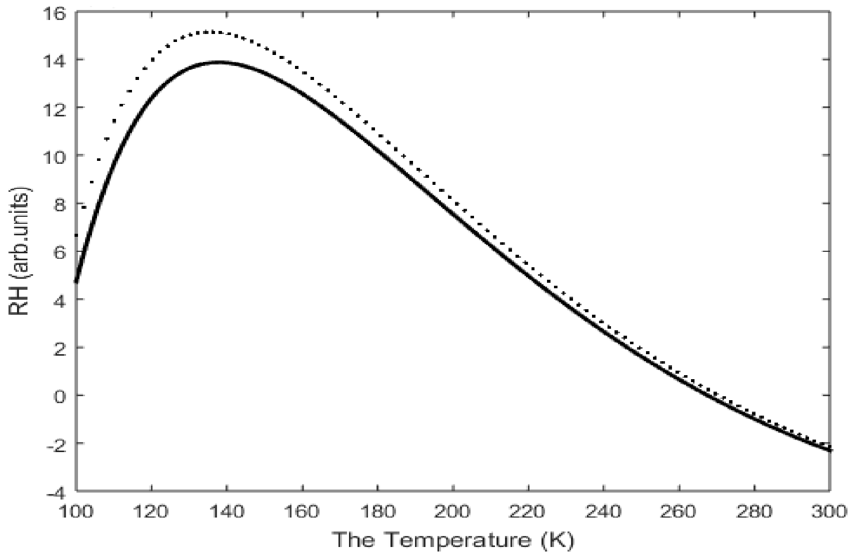


**Figure 1:** The dependence of the HC on the temperatures of system

The dependence of the quantum Hall coefficient on the temperature of the system in a cylindrical quantum wire with a parabolic potential is clearly shown in Figure 1. We can see that this dependence is nonlinear, after reaching the maximum value at about 135K, the quantum Hall coefficient decreases nonlinearly with increasing temperature. This can explain how chaotic thermal motion affects the quantum Hall effect, the stronger the chaotic thermal motion, the less likely the quantum Hall effect will occur. However, at too low a temperature, the mobility of the carrier is limited, and the quantum Hall coefficient also decreases.

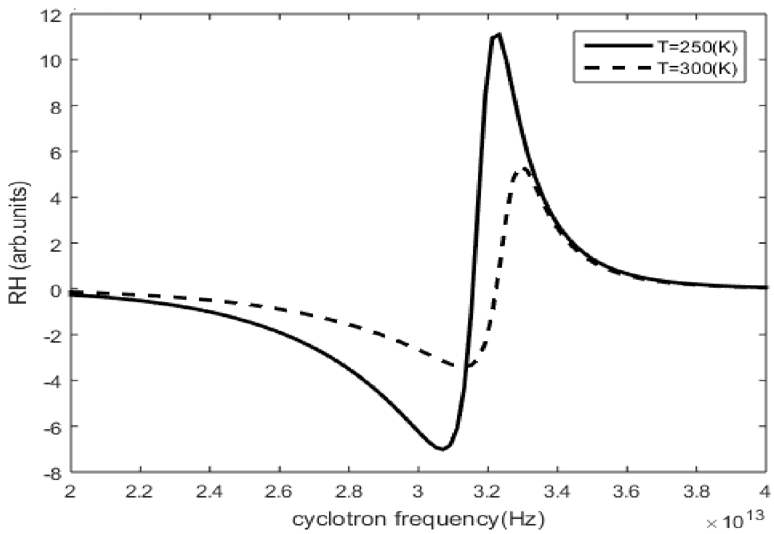


**Figure 2:** The dependence of the HC on the cyclotron frequency at different values of temperature T of the system

Figure 2 show that the dependence of the quantum Hall coefficient in a cylindrical quantum wire with a asymmetric parabolic potential on the cyclotron frequency of the magnetic field at different values of temperature. We can see that if the temperature of the system changes, the value of the cyclotron frequency corresponding to the maximum Hall coefficient also changes.

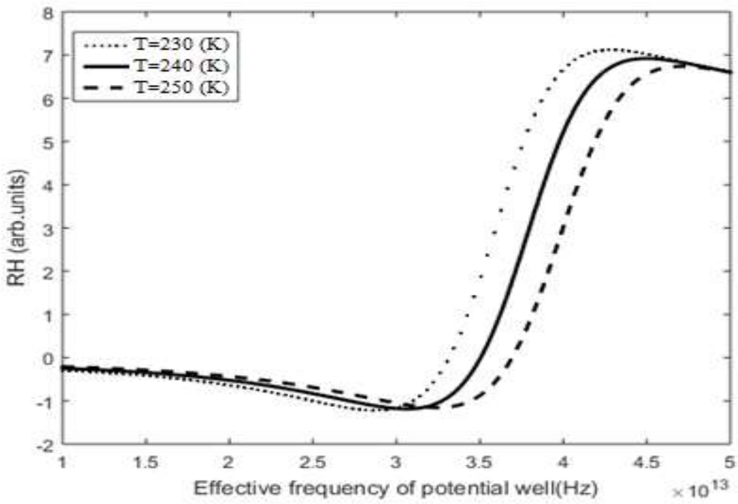


**Figure 3:** The dependence of the HC on the effective frequency of potential at different values of temperature T of the system.

We can see from Figure 3 that the effect of temperature on the dependence of HC on the effective frequency of the potential. The change in the temperature of the system also changes the values of the effective frequency of potential at which Hall coefficient has a maximum value

## 4. CONCLUSION

In this paper, by using the quantum kinetic equation method, we have obtained analytical expressions for the Hall coefficient in Cylindrical Quantum Wire with asymmetrical parabolic potential for case of electron- acoustic phonon scattering. It can be seen from these expressions that the dependence of the the Hall coefficient in Cylindrical Quantum Wire with asymmetrical parabolic potential on the temperature T is nonlinear.

The analytical results are numerically evaluated and plotted for a specific quantum wires GaAs/AlGaAs to show clearly the dependence of Hall conductivity on the temperature T of the system. Numerical results show that this dependence is nonlinear, after reaching the maximum value at about 135K, the quantum Hall coefficient decreases nonlinearly with increasing temperature. This can explain how chaotic thermal motion affects the quantum Hall effect, the stronger the chaotic thermal motion, the less likely the quantum Hall effect will occur. However, at too low a temperature, the mobility of the carrier is limited, and the quantum Hall coefficient also decreases. In addition, the change in the temperature of the system also changes the values of the cyclotron frequency and the effective frequency of potential at which Hall coefficient has a maximum value.

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