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Mapping of Mathematical Problem Solving Processes in Middle School Students: A Methodological and Analytical Approach

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ABSTRACT

Problem solving is now, one of the major skills of the 21st century. In this context, the STEM (Science, Technology, Engineering and Mathematics) approach goes beyond the teaching of school subjects, and the teaching-learning of mathematics should allow learners to appropriate skills of problem solving and solution finding in different life situations.

In fact, solving mathematical problems generates cognitive, metacognitive, affective and emotional processes. Therefore, the visualization of these processes allows teachers to implement adequate pedagogical strategies aimed at the development of learners' abilities and skills to solve mathematical problems. In the context of a digital pedagogy, its allowing the designers of computer systems to model the processes in order to set up intelligent tutorial systems (expert systems) helps in the development of mathematical problem solving skills.

We present in this article a methodological and analytical approach allowing to verbalize, visualize the networks of resolution of mathematical problem and to represent them schematically. We conclude with a modeling of these networks.

KEYWORDS: Mathematics, cognitive process, problem solving, thinking aloud, resolution network modeling.

I. INTRODUCTION

Problem solving is omnipresent in our daily life. It is now one of the major skills of the 21st century, the age of the Internet and economic globalization. It is not surprising, therefore, to see all educational systems insisting on the integration of problem solving as a teaching object or as a tool for in-depth learning and skill enhancement (Mudaly, 2021), or to see that the Organisation for Economic Co-operation and Development (OECD) and the PISA program (Programme for International Student Assessment) have set problem solving as a major objective (Albay, 2020).

Problem solving plays a key role in mathematics learning (Wilson, Fernandez & Hdaway, 1993). It is even at the heart of teaching-learning mathematics, "Doing mathematics is solving problems" (Fagnant & Vlassis, 2010). In the context of STEM (Science, Technology, Engineering and Mathematics), problem solving is an important lever in the educational innovation of the 21st century (Amalina & Vidàkovich, 2022).

Mathematics problem solving involves cognitive constructs (declarative knowledge, procedural knowledge, strategies, algorithms...), metacognitive constructs (planning, self-regulation, evaluation..), and affective constructs (motivation, attitudes toward mathematics..) (Aurah, Cassady & McConnell, 2014). Mathematic problem solving is therefore a complex cognitive process occurring in the learner's memory.

However, learners may use algorithms when solving problems without having a clear idea of the concepts, rules, properties, etc. It turns out that verbal visualization (representation using a natural language) or schematic visualization (semi-formal representation) of such a process can be very useful. It allows to detect and identify the nature of errors and the levels of cognitive and metacognitive obstacles encountered by the learners during the problem solving tasks.

In a computer context, the modeling of solving processes allows designers and developers of computerized learning systems to develop intelligent tutorial systems that help develop mathematical problem solving skills.

Johnstone & El-Banna (1986) and Longo, Anderson & Wicht (2002) have presented methodological approaches based on cognitivism, consisting in representing the problem solving networks generated by students during the solving of given problems.

The objective of this article is to implement a mapping of the resolution processes (resolution network) generated by middle school students when solving a mathematics problem (arithmetic problem) by implementing and experimenting one of the most used techniques in this kind of research: Thinking Aloud.

II. LITERATURE REVIEW

A. Problem and Problem solving in the psychological field

According to Duncker (1945), a problem exists when an individual has a goal to achieve, but does not know how to get there. According to Polya (1973) problem solving in mathematics is a cognitive process developed by a learner when faced with a situation where the goal to be reached is not immediate.

The resolution of a mathematical problem corresponds to the construction and manipulation in the learner's mind of a mental model (Bodner & domin, 2000) (internal representation of the process) which depends on a large base of cognitive and metacognitive knowledge (declarative and procedural knowledge, strategies, algorithms, techniques, approaches, motivation, attitudes, language...) This theory of mental model is one of the most influential theories of cognitive psychology in research on problem solving (Johnson Laird, 2000).

Several factors can influence the resolution of a problem: first of all, memory plays a determining role in the construction and manipulation of such a model (Solaz-Portolés & Lopez, 2007). The availability of pre-acquired knowledge necessary for the resolution. Brown & al (1989) then Lave & Wegner (1991) raise the determining role of cognition. Schoenfeld (1992) shows the importance of representations in the action of solving, by distinguishing between the search for information relevant to the resolution in the long term memory (facts, concepts, principles, properties, analogies, generalizations, strategies, approaches, algorithms, attitudes. and the way in which this information will be used during the resolution (appropriation of the context and the goal to be reached, identification of the information given by the problem and given by the memory, modelling of the problem, transformation of the data in the working memory, validation of each step of the resolution..), the linking of the information, the ordering of the hypotheses.

Daniela (2015) points out that the resolution of mathematics problems by a learner appeals to the abilities of the learner in memorization, perception, reasoning, conceptualization, appropriation of language, but also to these emotions, motivation, self-confidence. Bakar & al (2020) point out that, solving mathematics problems require metacognitive skills: the ability to plan, self-regulate, evaluate and reflect, which is confirmed by the work of Velasquez & al, (2019).

Agreeing with Ton de Jong, solving a mathematics problem implemented two domains:

- The domain of the problem: its context, its data, the question to be solved (goal to be achieved);
- The domain of the cognitive process: the reading of the problem, its appropriation, the pre-requisite knowledge, the strategies, the approaches, the algorithms, the planning...

In conclusion, we can use the model of Frensh & Funke, (1995) to model a mathematical problem solving situation in black box (figure 1). Solving a mathematical problem corresponds to reading the problem (data and context), identifying the goal to be reached (solution or answer to the question) and setting up a set of operations or algorithms and tools allowing to overcome the barriers or obstacles to the solution.

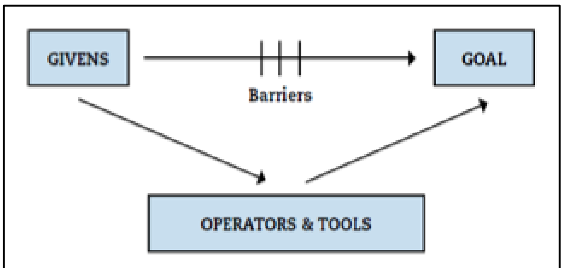

Figure 1: Modeling the problem situation

(Frensch, P.A. and J.Funke; 1995)

B. Visualization and mapping of problem solving processes: what it is and how to do it?

Since the emergence of cognitivist theories, researchers in learning psychology, education and recently in pedagogical engineering have been increasingly interested in the internal structures and processes of learning, in order to better understand the mechanisms of comprehension, analysis... and problem solving, with the aim of developing the most adequate and adaptive teaching-learning devices.

The literature on the study of problem solving processes mentions several verbalization techniques, such as questionnaires (Lorenzo, 2005), process mining methods (Glodhammer & Barkow, 2017).

The oral verbalization remains one of the methodological approaches widely used in research to account for the processes of problem solving and the blow of the internal model. This approach is based on the "Think aloud" technique. Thinking aloud is a qualitative technique in which the learner is asked to solve a problem by thinking aloud to report on the process of solving it (De jong, 2005). According to Van Someren et al (1994), the thinking aloud method is a practical approach to modeling the cognitive processes generated by a learner during problem solving.

In fact, thinking aloud is not a recent strategy. It stems from the principles of task analysis put forward by John B. Watson in the 20th century, and later developed by Ericsson and

Simon (1980; 1984) and Ericsson 2006). The "think aloud" method has been used for different purposes. Gresty et al (2005), used this technique to evaluate an online resource. Very recently, this method has been used by P.Sopamena (2018) and then by V.Mudaly (2021) to visualize and report on students' path (generated cognitive processes) when solving mathematics problems.

III. RESEARCH FOCUS

In this research, we are interested in verbalizing and reporting on the different stages and cognitive processes generated by a sample of middle school students during the resolution of a mathematical problem (algebraic problem). Our objective is to trace the student's resolution network, to analyze it and then to model the resolution path by limiting ourselves to the navigation between the problem data (problem domain and context), the pre-required data (long-term memory domain) and the data resulting from the different transformations (working memory domain).

IV. METHODOLOGICAL APPROACH

A. Selection of the student sample

The 20 students participating in the interview session were enrolled in the third year of secondary school in the 2021-2022 school year, selected according to gender (10 girls and 10 boys), level in mathematics (considering their test scores during the year; their motivation to participate; their level of language and verbal communication. They were informed that this assignment would not be graded so that they would not stress about the grade and could use their own methods and say whatever they thought. The students are grouped into two groups:

- Group SHP: these are the high achievers in mathematics with averages above 15 /20, possessing a good level of language.
- Group EPP: students with low performance in mathematics who have obtained averages a score between 10 and 15 /20, with an average level of language (sometimes they have to speak in Arabic to say how they think).

B. Choice of problem situation

- "I am 23 years younger than your age. In 15 years, your age will be triple the age I am. What is my age? What is your age? Can we say that you speak with an adult?"

The problem that was the subject of the experiment was taken from the 3rd grade college textbook of the Moroccan curriculum. It is the typical problem of the didactic module "Algebra", a module characterized by the presence of three aspects (Kieran, 2007): generative (expressions, equations, inequations, identities...), transformational (mathematization of the different objects by determining and analyzing linguistic and conceptual data) and global (solving problems of generalizations, equation setting, modeling...) Natural language (language code) is a determining factor in the perception and appropriation of the chosen problem situation (the use of tenses and personal pronouns).

C. Data collection method

We used the think aloud technique by inviting each student to say aloud, in as much detail as possible, how he/she solves the problem. In order to further investigate the cognitive and metacognitive processes generated by each student during the problem solving process, we used a complementary technique: the semi-structured interview. For the choice of questions, we were inspired by the questions of the model presented by Polya (1973) in his book "How to solve it". We also considered the students' written records as a source of data.

The transcripts recorded during the thinking aloud, completed by the data from the written traces of each student, are converted into a process diagram (or network map), tracing the path of the data between the 3 domains concerned: problem domain, long-term memory domain, transformation domain.

V. RESULT AND DISCUSSION

The analysis of the problem situation allowed us to identify the three relevant problem domains (problem data, memory data, and transformation data) that we group together in Table 1 to form our framework.

Table 1: The different data and categorization generated during the problem solving (frame of reference)

Problem data	*Memory data*	*Transformation data*	*Solution*
Dp1 : I have	*DM1 : the present of the verb to have, the personal pronoun "me"*	*DT1 : let x be my age*	*S1*
Dp2 : your age	*DM2 : I am shorter than you by 23 years*	*DT2 : your age is (x + 23) years old*	*S2*
Dp3 : 23 younger than your age	*DM3 : The age will increase by 15 years*	*DT3 : (x +23) + 15*	*S3*
Dp4 : in 15 years	*DM4 : The verb to have*	*DT4 : (x +23) + 15 = 3x*	

	in the future		
Dp5 : will have	***DM5 : triple is three times***	***DT5 : x + 23 + 15 = 3x***	
DP6 : triple your age	***DM6 : If a parenthesis is preceded by a + sign, you can remove the parentheses without changing anything***	***DT6 : x + 38 = 3x***	
Dp7 : that I have	***DM7 : the rule of the sum of two relative numbers***	***DT7 : -x + 3x = 38***	
	DM8 : si x + b = ax alors ax – x = b	***DT8 : 2x = 38***	
	DM9 : ka + kb = k(a+b)	***DT9 : x = 38/2***	
	DM10 : si ax = b then x = b/a		
	DM11 : age difference		
	DM12 : from the age of 14 an adult can have children		
	DM13 : a/b = (a/k) / (b/k)		

The resolution network maps of the students in each group are overlaid to derive an overall model map of each group. In this section we present the two resolution network models (or resolution process maps) obtained from the two student samples. The SHP map representing the process model generated by the students of group A (High Performers) and the SLP map representing the process model generated by the students of group B (Low Performers).

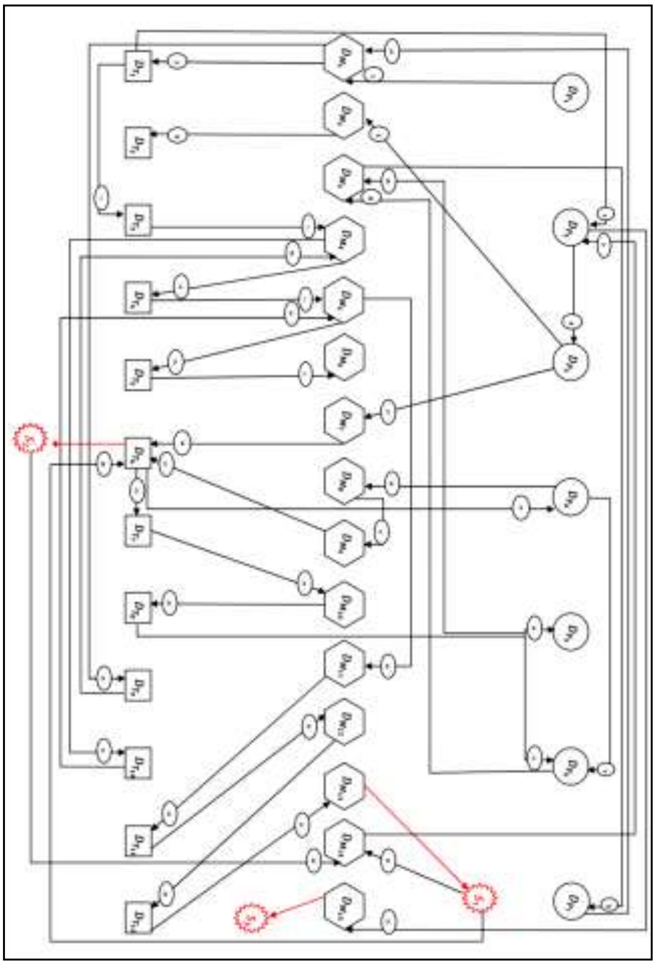

Figure 2: SHP Student Resolution Network Model

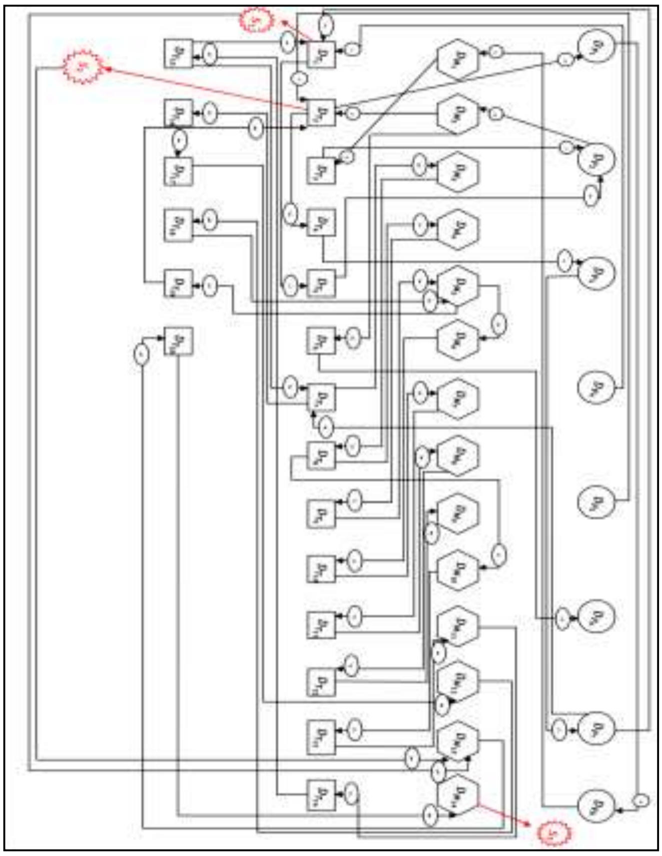

Figure 3: SLP Student Resolution Network Map Model

One SHP student began her cognitive process of solving by citing Polya's four processes of mathematical problem solving and then said:

"I will read the text word by word trying to understand the mathematical meaning of each piece of data and determine the main data of the problem".

After the first reading she said

"To understand the text well, I will apply this problem to myself. I am 23 years younger than you, my teacher, that is to say I am 23 years older than you".

The students in this group are able to read the problem by identifying this data. Some of the students in the group do not give importance to the data "I have".

Then they move on to the second step to design a plan by trying to translate and transform the information and linguistic data into mathematical notation and equations. However, most of them had difficulty translating the word "minus" and they did not do the transformation correctly, which led to a blocking or a non significant result (negative age), but they recognized their mistake.

This phase of conscious error allowed the students to resume the course, by reading the problem more carefully, and by adopting a more concrete approach

"15" in mathematical notation because of language and also concentration problem, which she friends after the execution of the plan that she established, in a state of blocking by finding that the age of the speaker is negative, by saying " it is impossible, I found that 2 x = - 23 that is to say x is a negative number on the other hand the age is always a positive number, therefore I made errors somewhere! During the execution of the plan, SHP detected that there was an error by realizing that age is a positive number, which shows her intelligence and her skills, which allowed her to regulate the course by going back to the previous steps trying to detect the error.

"I found my mistake, I am shorter than you by 23 years, so if my age is x years then yours is x + 23 years", then she continued reading and said "In15 years that is to say after 15 years" a silence of 11 seconds after she said "after 15 years, I will not have the same age, if I am now 14 years old after 15 years I will have 14 + 15 that is 29years yes so after 15 years I will have 15 + x and your age and x + 23 + 15 that is to say x + 38 years".

The detection and correction of errors was a very interesting step in the resolution, since it allowed to translate the words (natural language) into mathematical notation (formal language) and, to find the key links between the different data, to conceive a good plan and to find the sought solution.

"In order to answer the last question I have to calculate the difference in ages which is 23 years according to the statement, which makes sense, because a 23 year old adult can have children so the answer is yes may be speak with an adult".

When analyzing the solving process of the students in the SHP group, we notice that most of the students manage to align themselves after a moment of doubt or error with Polya's (1973) heuristics. They manage to identify the data (85.71%), to restore the relevant data from long-term memory (92.30%) and to make the necessary transformations (77.78%).

Overall, the students in this group used up to 42 navigations between the three domains involved before arriving at the solution. However, some students arrive at the solution by taking shortcuts and consequently reduce the number of cognitive processes needed to solve.

Error detection and path regulation by modeling the situation, rereading the statement, determining all the data of the problem and giving examples to understand and avoid blocking show that the SHP students have highlighted very important cognitive and metacognitive strategies and that they just needed work in the comprehension phase more precisely, the determination of the data of the problem and in the phase of translation and transformation of linguistic information into mathematical notation, algorithm and equation.

In the case of the group of SLP students, they begin the cognitive process by reading the statement and imagining the presence of the person "your"

"I have two unknowns: my age and his age so I have to look for the equations of the system, then solve the system and find the solution".

For them it is a problem already seen, and that it is reduced to the resolution of a mathematical equation.

"I have faced this problem before. It's about solving a mathematical equation and maybe a system of equations.

They start by reading the problem, identifying the data in the problem, for them just the numerical data.

They start by reading the problem, identifying the data of the problem, for them just the numerical data. "The word me, it's not a data, it's just to understand the problem".

The students therefore fail to identify all the data, and yet they seek to determine a system of equation by seeking to establish links between these data in complete uncertainty.

"I remember that we have to look for x and then replace it to find y, so I'm going to sum the two equations member by member.

"It is impossible, the age must be an integer and I found that x=23/4, so my method is false.

The analysis of the EPP students' solve aloud protocols shows that they do not master the steps of the problem solving. They find it very difficult to identify the data of the problem. They apply notions and relations that they do not master

The superposition of the different resolution network maps put into play by the students allowed us to build a model of explicit data flow (figure 4) between the three domains involved in the resolution of a problem.

This model is made up of three domains:

- the domain of the problem situation which includes the context and the data (numerical, linguistic, iconic, symbolic... ;
- The domain of declarative knowledge stored in the long-term memory (LTM) (vocabulary, definitions, properties, relations, rules, algorithms, approaches...) These data are concerned by processes of research and restitution.
- The domain of transformations that take place at the level of the working memory (WM) (translations, conversions, applications of rules and procedures, verifications...).
- The different interactions between the three domains, which generate the different processes of the resolution

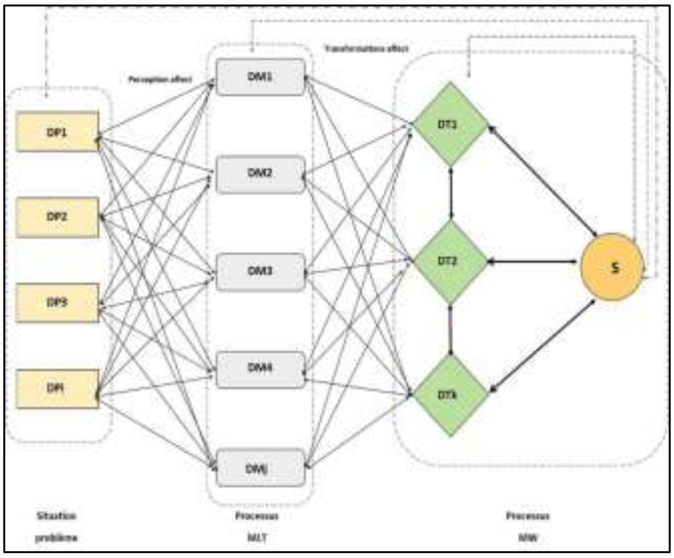

Figure 4: Modeling the network for solving a mathematics problem

VI. CONCLUSION

The in-depth analysis of the theories modeling learning and of the literature specialized in the study and analysis of the problem of teaching and learning problem solving as a tool or object of mathematics teaching ; Our observations and practices of mathematics teaching, as well as this study and its results have shown the interest of modeling for the student and for the teacher since it allows to trace the network of processes of the resolution of each student in front of a problem situation and to know his way of thinking and to recognize the different difficulties and the points of blocking in order to intervene.

As far as the teacher is concerned, it allows him/her to choose the right situation and to ask questions before developing a problem situation: what knowledge and skills does he/she need to solve the problem? Does the student have them? What knowledge and skills can the student acquire after solving this problem? This is in order to achieve the desired objectives of the problem solving.

We are aware of the difficulties of this work, which is tedious, especially since we need a large number of students in the collection and analysis of data, which means a large number of interviews, transcripts, network maps. All this poses a problem of the time that is counted in the teaching and also the difficulty for some students to organize their logical thinking, especially for those students who find some perplexed to verbalize what they think in front of a problem, which poses a real challenge to follow their progress, especially when the number of problem demands increases and when other non-cognitive factors (psychological: motivation, confidence, concentration, self-efficacy...) influence the performance in solving mathematical problem.

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